MEASUREMENTS & INSTRUMENTS

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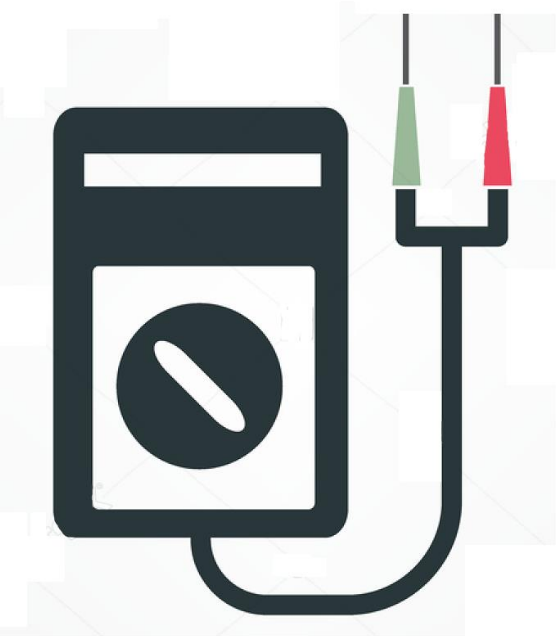

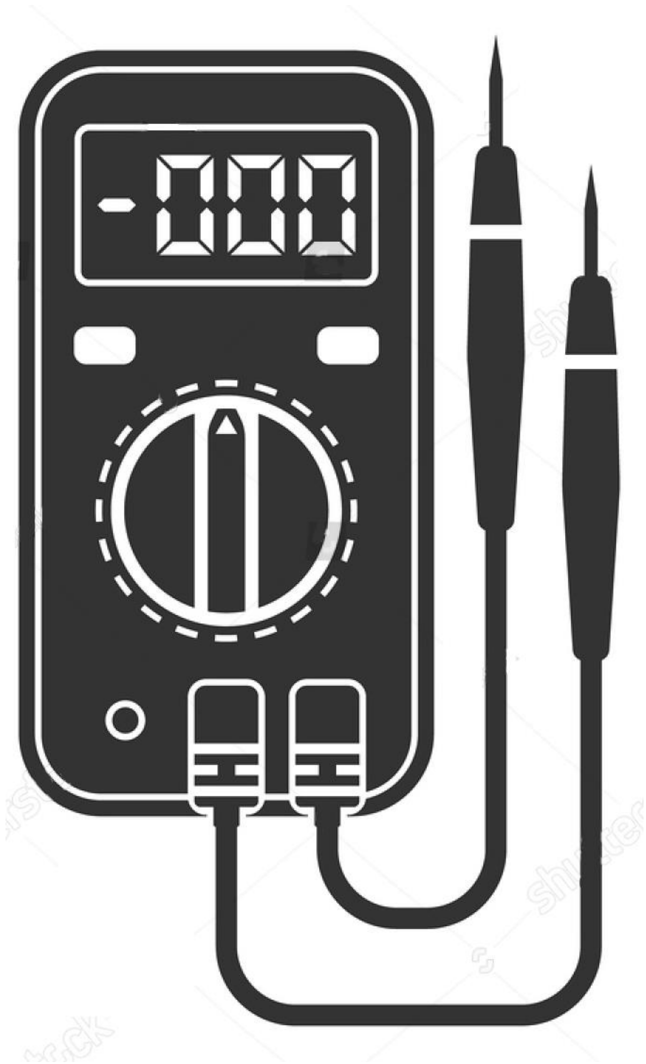

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Second stage, 2017 – 2018

- Research has shown that students understand more from a lecture if they write things down. Keep focus and ask if you did not understand.

- Study to Learn …… Not just to pass an exam.

- Your attendance is important to you, not for me. Make sure you sign the attendance sheet.

- Homework's are crucial, 10% of your mark depends on them.

- Do not fully depend on me, go and search the YOUTUBE and GOOGLE.

- Try to understand why are you studying this material, is there any relationship between this material and the real world.

Syllabus :

Chapter 1: Measurements and error
Error and its classification
Chapter 2: Units and standard SI system
Chapter 3: Classification of instruments
Chapter 4: Various measurements methods for determining resistance, inductance, and capacitance
Chapter 5: Various measurements methods for determining frequency, phase angle, and power factor
Chapter 6: Various measurements methods for determining hysteresis loop
Chapter 7: High voltage measurements and testing
Chapter 8: Cathode Ray Oscilloscope (CRO) and Digital Storage Oscilloscope (DSO)
Chapter 9: Lissajous Patterns

Chapter 1
Measurements and error

1-Introduction

The development of science and technology is very much dependent upon a parallel development of measurement techniques. Knowledge largely depends on measurement, and the technology of measurement called instrumentation, serves not only science but all branches of engineering, medicine and almost every spheres of human life. Measurement instruments are used in monitoring and control of processes and operations. Most specialized instruments are used in experimental scientific and engineering work.

Measurement methods may be classified into direct and indirect methods. In direct methods of measurement the quantity to be measured is compared directly against a standard of same kind of quantity. The magnitude of the quantity being measured is expressed in terms of a chosen unit for the standard and a numerical multiplier. A length can be measured in terms of metre and a numerical constant. Thus, a 10 meter length means a length ten times greater than a metre.

Direct methods of measurements are though simple, it is not always possible, feasible and practicable to use them. The involvement of man in these methods makes them inaccurate and less sensitive. For these reasons, engineering applications uses measurement systems which are indirect methods of measurement.

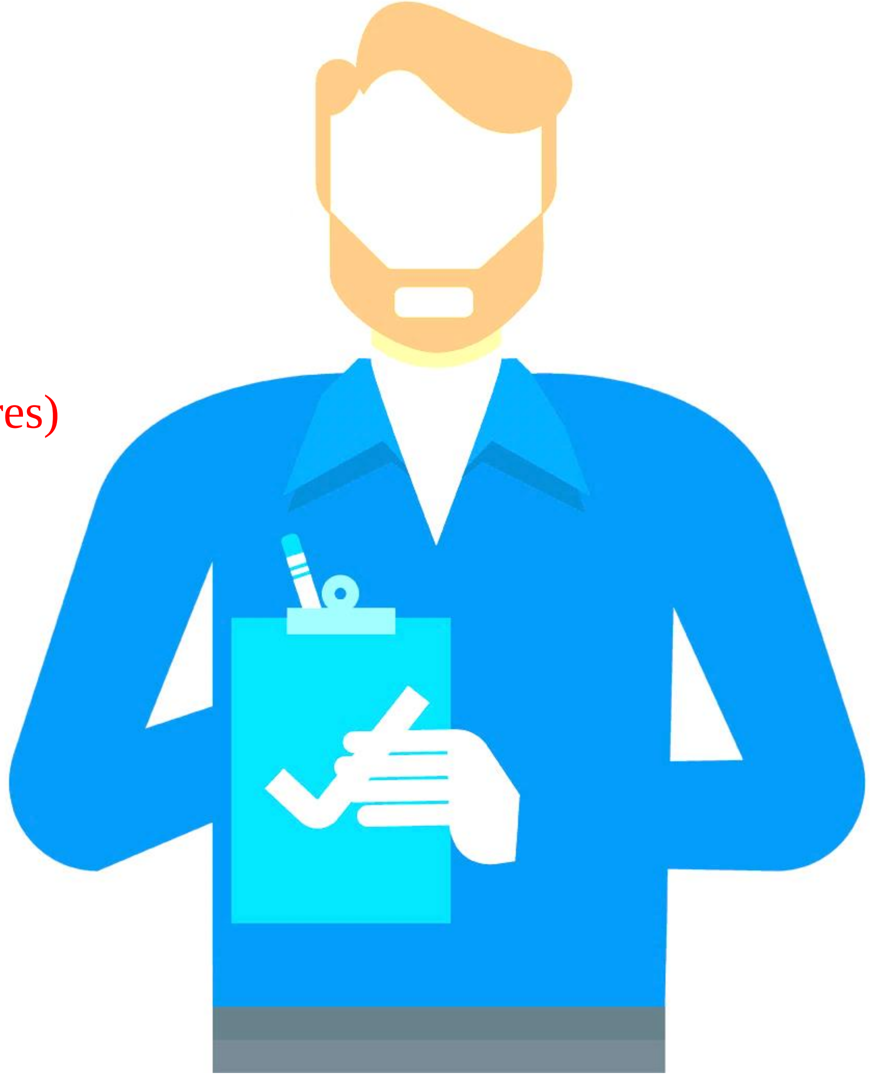
A current is measured by an ammeter which gives a deflection of a pointer on a scale corresponding to the current. Thus, the current is not compared with a standard current, rather it is converted into a force which causes the pointer to deflect.
An electrical signal is a versatile quantity because of the fact that it can be easily amplified, attenuated, measured, rectified, modified, modulated, transmitted, and controlled. This fact created the interest to use electrical methods to measure non-electrical quantities. For this purpose a device known as a transducer is used to convert to non electrical quantities into electrical quantities. For example Microphones. A microphone is a transducer that takes sound energy and converts it into electrical energy. It takes some type of sound as its input and converts it into electrical pulses or signals.

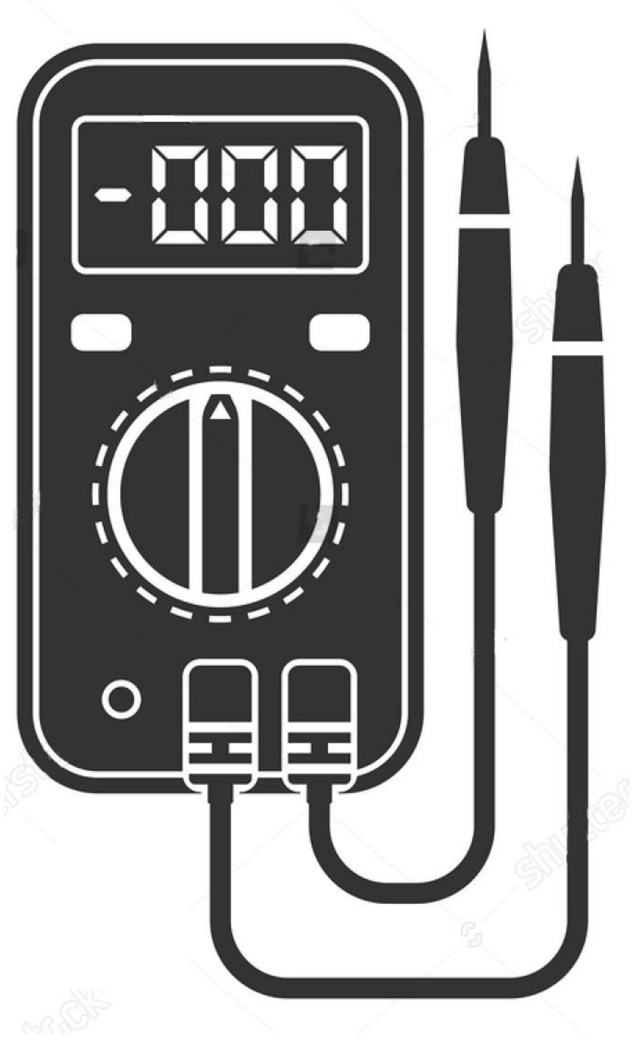
The electronic instruments, now-a-day, are computing, manipulating, and processing information in much the same way as the mind. For these reasons, the important of studying electronic instruments is increased.

Increases in availability and types of computer facilities, and decrease in the cost of various modules required for digital systems are accelerating the development of digital instrumentation for the measurement. The digital form of measurement is also used to display the measured quantity in readable numbers instead of a deflection of a pointer on a scale which completely eliminates a number of human errors.

2-Notes on what should be focused on when taking a measurement.

- Choice of measurement method
- The availability of materiel
- The calibration of instruments
- Time require
- Difficulties in measurement
- Record preparation
- Accuracy and precision (Significant figures)
- Error in measurements





The necessary instruments that you are going to use in the practical session of this module are shown in figure (1).

Function generator

Digital Multimeter

DC Power supply

(CRO) Analog Oscilloscope

Figure (1) practical session electrical instruments

3-Measurement System Architecture

Figure (2) illustrates the block diagram of a typical measurement system. The quantity under measurement (QUM) is converted to a useful form, such as a voltage, current or physical displacement by an input transducer or sensor.

Note: QUM = quantity under measurement. S = sensor, A = amplifier, SCF = signal conditioning filter, DSO = digital storage oscilloscope, ATR = analog tape recorder, AAF = antialiasing (lowpass) filter, n1 = noise accompanying the QUM, n2 = noise from electronics, n3 = equivalent quantization noise, ADC = analog to digital converter, DC = digital computer, MON = monitor, KBD = keyboard.

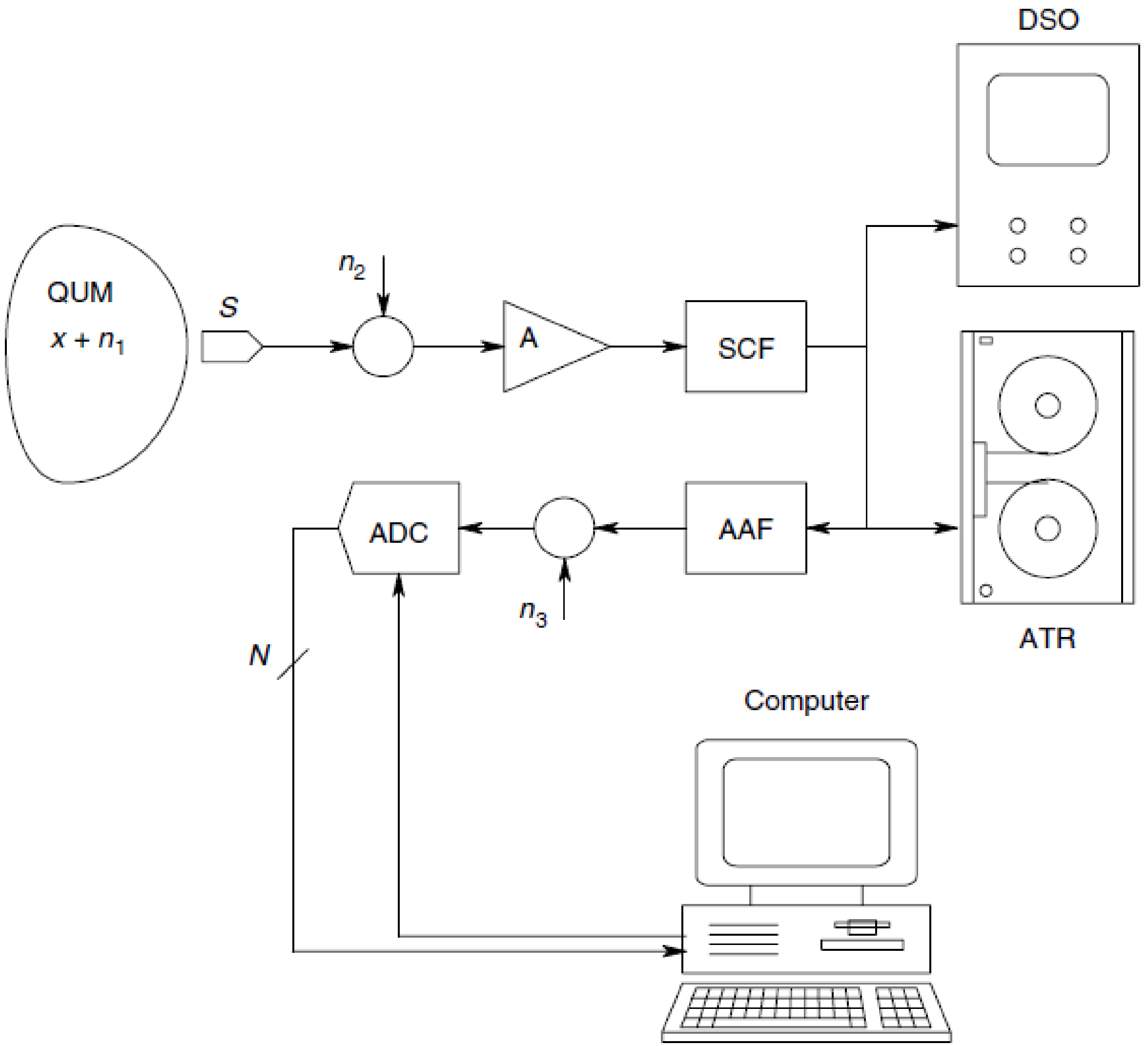


Figure (2) A generalized measurement system.

4-Error in measurements:

Though today we have very sophisticated measurements systems, we can not think of a measurement without error. The error can be reduced by selecting a proper method of measurement and by taking some necessary precautions at the time of measurement. Recording the measured data also plays an important role. By now it has become clear that a quantity can never be measured with perfect accuracy in practice. So it is necessary to know the limit of maximum possible error in any measurement. Measurements without this have no meaning. It has no meaning in saying that the resistance of a resistor is 100 ohm. But, if it is said that the resistance of a resistor is 100 ±2.5 ohm, then it means that the resistor can be used wherever a resistance of value varying from 97.5 ohm to 102.5 ohm can be tolerated.

Absolute error $(\in_\circ)$ is also called as “maximum possible error”
Error in measurement (δR) $\delta R = A_m - A$
Where A_m = measured value, A = accurate value
Absolute error $(\in_\circ)$ is the limit of error in measurement. In other words (δR) must never be higher than $(\in_\circ)$. Therefore, $|\in_\circ| = max \quad |A_m - A|$

Relative error $(\in_r)$ Absolute error does not give any information about accuracy. For example, -1 volt error in measurement of 1100 volt is negligible, but -1 volt error in measurement of 10 volt is never acceptable.

Relative error is the ratio of absolute error with the accurate value.

Therefore , relative error $\in_r = \frac{\in_\circ}{A}$

If $\in_\circ$ is negligibly small as compared to A_m, then equation above can be written as $\in_r = \frac{\in_\circ}{A_m}$

Generally, relative error is given in per cent of measured value,

Per cent error = 100 $\in_r$

Correction (δC) is negative or error.

Therefore, correction $\delta C = -\delta R$ $\therefore\ A = A_m + \delta C$

So addition of correction in the measured value gives accurate value.

Example 1.1
Homework

Static sensitivity

In general, sensitivity is defined as the ratio of the incremental output to the incremental input. When an input-output calibration curve is a straight line as that of figure (3-a) the static sensitivity of the instrument is the slope of the calibration curve, i.e the sensitivity is the ratio of output to input. If the calibration curve is not a straight line which is normally the case, the sensitivity is not constants, it will vary with the input as shown in figure (3-b). For a meaningful definition of sensitivity the output quantity must be taken as the actual physical output observed, not the meaning attached to the scale numbers. For an example, the actual physical output of a voltmeter is the angular deflection of the pointer and the unit of sensitivity, therefore, will be radian/volt.

Example 1.2

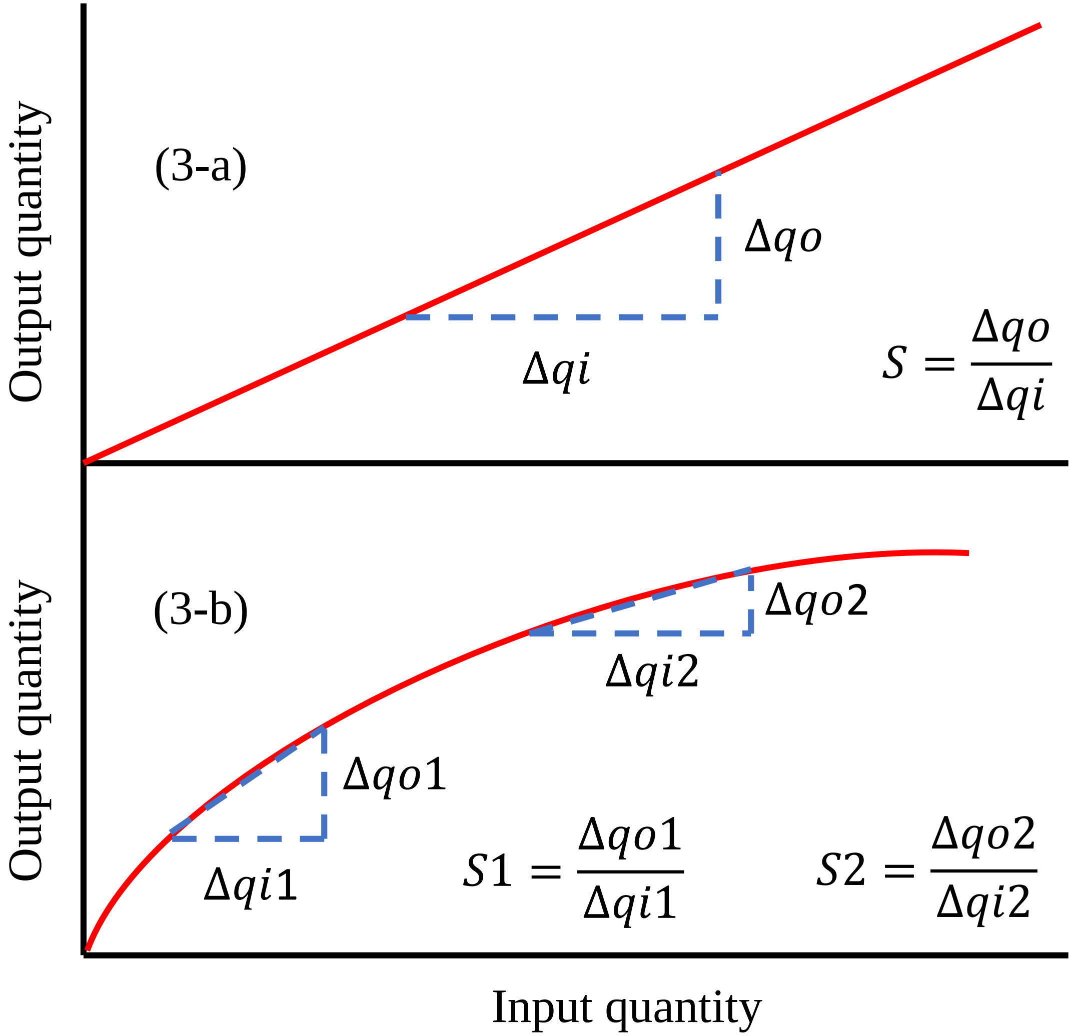


Figure (3-a) linear response, (3-b) non linear response

5-Error and its classification

It was mentioned that no measurement can be made without an error. But, it is important for the experimenter to take proper care so that the error is minimized. A little carelessness of the observer may introduce the gross error in measurements. Selection of a suitable instrument is very important. Thus, the human error may be the gross error in the measurement. Some of the errors may be attributed by the shortcomings of the instrument. Environmental conditions under which the measurement is being taken may introduce an error. These are errors which are random in nature and may be minimized by taking average of a large number of readings.

Thus we see that there are different sources of errors and generally errors are classified under the following three main groups:

A- Gross errors
B- Systematic errors
C- Radom or accidental errors

A- Gross errors

There are human errors which includes the errors caused by mistake in using instruments, recording data and calculating measurements results. A person may read a voltmeter indicating 101 V as 110 V. An ordinary wattmeter may be used (by mistake) to measure power in a low power factor load. A person may set a multimeter on DC voltage to measure an AC voltage as shown in figure (4). Or put the multimeter terminals in wrong holes.

V⎓ to measure DC Voltage

V~ to measure AC Voltage

mA to measure milliamps (Red)

20A to measure amps (Red)

COM, the negative terminal (Black)

VΩ→ to measure Voltage resistance and diodes (Red)

Figure (4) Multimeter and Gross errors

B- Systematic errors

These are inherent errors of apparatus or methods “errors in instruments”. These errors always give a constant deviation. On the basic of origins of errors, systematic errors may be divided into the following sub-groups:

1- Constructional error, a manufacturer always mentions the maximum possible errors in construction.
2- Error in readings or observation
   A) Construction of scale “division”
   B) Fineness and straightness of pointer
   C) Parallax, “without a mirror under the pointer”
   D) Efficiency of the observer
3- Determination error, it is due to “indefiniteness” in final adjustment of the measuring apparatus.
4- Natural error of method, none of method is perfect. Each method has some errors.
5- Errors due to other factors such as:
   A) Temperature variation
   B) Effect of time on instruments “ageing of instrument”
   C) Effect of external electrostatic and magnetic fields
   D) Mechanical error “springs, gearbox”

C- Radom or accidental errors

After corrections have been applied for all the parameters whose influences are known, there is left a residue of deviations. These are random errors and their magnitudes are not constant. Persons performing an experiment have no control over the origin of these errors.
These errors are due to so many reasons such as noise and fatigue in the working person. These errors maybe either positive or negative. Generally, these errors may be minimized by taking the average of a large number of readings.

6-Statistical analysis

- **The arithmetic mean**. The arithmetic mean of a number of readings gives the most probable value of a measured variable.

$$\bar{X} = \frac{X1 + X2 + X3 + \ \dots\dots\dots\dots \ + Xn}{n}$$

- **The average deviation.** A precise instrument will yield a low average deviation. The average deviation is defined as the average of the absolute values of the deviations of the readings.

$$d_{av} = \frac{|d1| + |d2| + |d3| + \ \dots\dots\dots + \ |dn|}{n}$$

- **The standard deviation.** The root mean square (r.m.s) deviation in statistical analysis, is known as standard deviation and is a very valuable aid.

$$\sigma = \sqrt{\frac{d1^2 + d2^2 + d3^2 + \ \dots\dots\dots . + dn^2}{n}}$$

- **The variance.** The variance V is defined as the square of the standard deviation.

$$V = \sigma^2 = \frac{d1^2 + d2^2 + d3^2 + \ \dots\dots\dots . + dn^2}{n}$$

Example 1.3
Homework

In case of measuring during a small intervals of time, or values are given in the form of frequency table, the frequency will be included in the equations as shown below.

$$\bar{X} = \frac{X1f1 + X2f2 + X3f3 + \ \dots\dots \ + Xnfn}{f1 + f2 + f3 + \ \dots\dots + fn}$$

$$\sigma = \sqrt{\frac{d1^2f1 + d2^2f2 + d3^2f3 + \ \dots\dots\dots. + dn^2fn}{f1 + f2 + f3 + \ \dots\dots + fn}}$$

$$d_{av} = \frac{|d1|f1 + |d2|f2 + |d3|f3 + \ \dots\dots \ + \ |dn|fn}{f1 + f2 + f3 + \ \dots\dots + fn}$$

$$V = \sigma^2$$

Example 1.4

- **Limiting errors**

Manufacturers of any equipment give guarantee about the accuracy of the equipment with some limiting deviations from the specified accuracy. Most of the measuring instruments are guaranteed for their accuracy with a percentage deviation of full scale reading. Components like resistors, capacitors, etc. are always guaranteed for the rated value with a tolerance in percent of the rated value. These limiting deviations from the specified values are called the limiting errors or guarantee errors. For example, if the resistance of a resistor is specified as 1 KΩ ±10% as shown in figure (5), the resistance of the resistor may have any value between 900 Ω to 1100 Ω.

Example 1.5
Homework

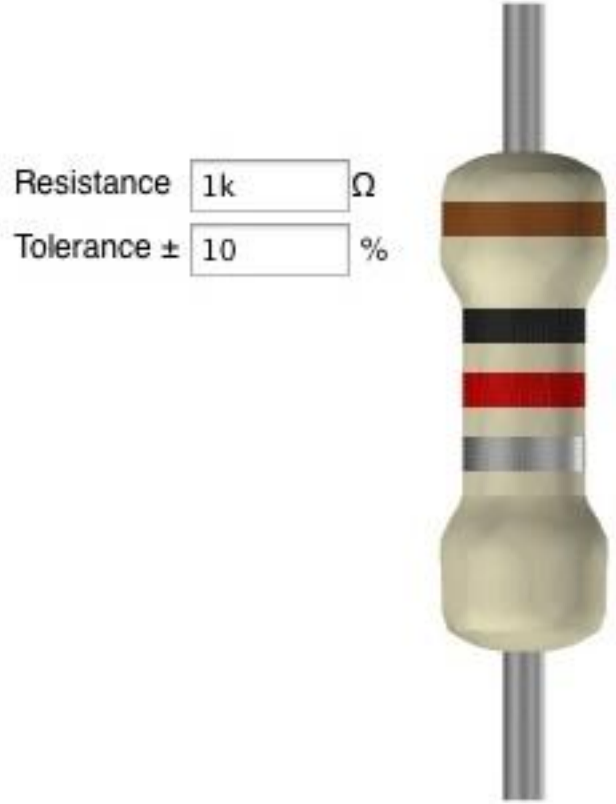


Figure (5) 1 KΩ ±10% resistor

Chapter 2
Units and standard SI system

1-Units

In the first stage of development of the technique of measurement, specific physical standards entirely unrelated to each other gave varying degrees of satisfaction. But With the development of science and technology of measurement a system of units with a logical and simple relation became desirable. A number of systems of electrical units have been used 1n electrical measurements at various times. Some of them are Only of historical interest, some are used chiefly in theoretical discussion, and some others are or have been employ in measurement and have been accepted for this purpose on either a national or international basis. So, it is in our interest to acquaint ourselves with some of important and commonly used systems.

2-Absolute units

A systems in which the various units are all expressed in terms of fundamental units is called an absolute system of units. Here the word absolute does not imply supreme accuracy rather it is used as oppose to relative. An absolute measurement does not compare the measured quantity with arbitrary units of the same kind, but are made in terms of some of fundamental units. The committee of the British Association on Electric Units and Standards, in formulating the absolute system of units in 1863, had expressed that the units should not be defined by a series of master standards, each defining one quantity in the way in which the units of length, mass and time are defined but that each electrical unit should be defined by some natural law which expresses the relation between the quantity concerned and the fundamental quantities of length, mass and time, for which internationally accepted, standards have already been established.

3-C.G.S. Electrostatic and Electromagnetic Systems

Before the M.K.S. system of units came in existence the most commonly used system of units in electrical works was the C.G.S. system of units. In this system the fundamental units of length, mass and time are centimetre, gram and second respectively. To define electrical units in terms of mechanical units, there are two convenient starting points. The expression for mechanical force between charges at rest (Coulombs law) may be used to develop an electrostatic system of units while the force equation between charges in motion (Electric currents) (Ampere's law) may be used to formulate an electromagnetic systems of units.

3.1-Electrostatic system:

1- Potential difference: Unit potential difference is the potential difference which must exit between two points in order that 1erg of work be done in moving unit charge from one point to the other.

2- Current: Unit current is defined as unit charge per second.

3- Capacitance: The Unit of capacitance of a body is the capacitance across which unit charge will cause unit potential difference to appear,

3.2-Electromagnetic system

1- Magnetic pole: A unit magnetic pole is that magnetic pole which, when placed 1 cm apart from an equal magnetic pole in free space, will exert a force of l dyne on each other.

2- Magnetic field strength: A unit magnetic field strength is that magnetic field strength which exerts a force of l dyne on a unit magnetic pole placed in the magnetic held.

3- Resistance: A unit resistance is that resistance in which l erg of energy is expended per second when unit current passes through it.

4- Potential difference (e.m.f): A unit potential difference is the potential difference across a unit resistance when unit current is passed through it.

5- Magnetic flux: A unit magnetic flux is that magnetic flux, the removal of which from a circuit of one turn induces an average e.m.f. of one e.m- unit. The time of removal being one second.

4-Dimensions of Mechanical quantities

- $Velocity = \frac{Length}{Time} \Longrightarrow V = \frac{L}{T}$
- $Acceleration = \frac{Velocity}{Time} \Longrightarrow a = \frac{V}{T} \Longrightarrow a = \frac{L}{T^2}$
- $Force = Mass * Acceleration \Longrightarrow F = M * a \Longrightarrow F = \frac{M*L}{T^2}$
- $Work = Force * Distance \Longrightarrow W = F * L \Longrightarrow W = \frac{M*L^2}{T^2}$
- $Power = \frac{Work}{Time} \Longrightarrow P = \frac{W}{T} \Longrightarrow P = \frac{M*L^2}{T^3}$

5-Dimensions of Electrical quantities

There are two systems of electrical units, the e.s. system and e.m system

5.1-Dimensions in (electro-static) e.s system:

Note: $r = \ distance\ between\ two\ charges, and\ \ \in is\ the\ permittivity$

- $Charge\ (Q)$ $\quad F = \frac{Q1Q2}{\in r^2} \Rightarrow F = \frac{Q^2}{\in L^2} \Rightarrow \frac{ML}{T^2} = \frac{Q^2}{\in L^2} \Rightarrow Q = \frac{1}{T}\sqrt{\in ML^3}$
- $Current\ (I)$ $\quad I = \frac{Q}{T} \Rightarrow I = \frac{\frac{1}{T}\sqrt{\in ML^3}}{T} \Rightarrow I = \frac{1}{T^2}\sqrt{\in ML^3}$
- $Potential\ difference\ (V)$ $\quad V = \frac{W}{Q} \Rightarrow V = \frac{\frac{ML^2}{T^2}}{\frac{1}{T}\sqrt{\in ML^3}} \Rightarrow V = \frac{1}{T}\sqrt{\frac{ML}{\in}}$
- $Capacitance\ (C)$ $\quad C = \frac{Q}{V} \Rightarrow \frac{\frac{1}{T}\sqrt{\in ML^3}}{\frac{1}{T}\sqrt{\frac{ML}{\in}}} \Rightarrow C = L \in$
- $Resistance\ (R)$ $\quad R = \frac{V}{I} \Rightarrow R = \frac{\frac{1}{T}\sqrt{\frac{ML}{\in}}}{\frac{1}{T^2}\sqrt{\in ML^3}} \Rightarrow R = \frac{T}{L\in}$
- $Inductance\ (\iota)$ $\quad \iota = \frac{V}{di/dt} \Rightarrow \iota = \frac{V}{\frac{I}{T}} \Rightarrow \iota = \frac{\frac{1}{T}\sqrt{\frac{ML}{\in}} * T}{\frac{1}{T^2}\sqrt{\in ML^3}} \Rightarrow \iota = \frac{T^2}{L\in}$

5.2-Dimensions in (electro-magnetic) e.m system:

Note: $r = \text{ distance of separation}, m1, m2 \text{ are pole strength, and } \mu \text{ is the permaeability}$

- $Pole\ Strength\ (m)$ $$F = \frac{m1m2}{\mu r^2} \Rightarrow F = \frac{m^2}{\mu L^2} \Rightarrow \frac{ML}{T^2} = \frac{m^2}{\mu L^2} \Rightarrow m = \frac{1}{T}\sqrt{\mu M L^3}$$
- $Current\ (I)$ We know that the force exerted upon a magnetic pole, of strength h m units and placed at the centre of a circular wire of radius r, due to a current I flowing in an arc of the circle of length L is given by:

$$F = \frac{mIL}{r^2} \qquad I = \frac{Fr^2}{mL}$$

$$I = \frac{FL^2}{mL} \Rightarrow I = \frac{\frac{ML}{T^2}L^2}{\frac{1}{T}\sqrt{\mu M L^3}\,L} \Rightarrow I = \frac{1}{T}\sqrt{\frac{ML}{\mu}}$$

- $Magnetic\ field\ strength\ (H)$ $$H = \frac{F}{m} \Rightarrow H = \frac{\frac{ML}{T^2}}{\frac{1}{T}\sqrt{\mu M L^3}} \Rightarrow H = \frac{1}{T}\sqrt{\frac{M}{\mu L}}$$
- $Charge\ (Q)$ $$Q = I\,T \Rightarrow Q = \frac{1}{T}\sqrt{\frac{ML}{\mu}} * T \Rightarrow Q = \sqrt{\frac{ML}{\mu}}$$
- $Potential\ difference\ (V)$ $$V = \frac{W}{Q} \Rightarrow V = \frac{\frac{ML^2}{T^2}}{\sqrt{\frac{ML}{\mu}}} \Rightarrow V = \frac{1}{T^2}\sqrt{M\mu L^3}$$

- $Capacitance\ (C)$ $$C = \frac{Q}{V} \Longrightarrow C = \frac{\sqrt{\frac{ML}{\mu}}}{\frac{1}{T^2}\sqrt{M\mu L^3}} \Longrightarrow C = \frac{T^2}{\mu L}$$
- $Resistance\ (R)$ $$R = \frac{V}{I} \Longrightarrow R = \frac{\frac{1}{T^2}\sqrt{M\mu L^3}}{\frac{1}{T}\sqrt{\frac{ML}{\mu}}} \Longrightarrow R = \frac{\mu L}{T}$$
- $Inductance\ (\iota)$ $$\iota = \frac{V}{di/dt} \Longrightarrow \iota = \frac{V\ T}{I} \Longrightarrow \iota = \frac{\frac{1}{T^2}\sqrt{M\mu L^3} * T}{\frac{1}{T}\sqrt{\frac{ML}{\mu}}} \Longrightarrow \iota = \mu L$$

Homework, practise the derivation

6-M.K.S. System(Giorgi System)

This System was first suggested by the Italian physicist, Prof. Giorgi, in 1901 and is known as the Giorgi-MKS. System. This system used the metre, kilogram and second as fundamental mechanical units instead of the centimetre, gram, and second of C.G.S. system. The system was adapted by IEC (International. Electrical Commission) at its meeting in 1938. The commission recommended the permeability of free space with the value of $\mu_0 = 10^{-7}$as the forth unit connecting the electrical to the mechanical units.

The advantages of the M.K.S. system over the C.G.S. system are:
1- Its units are identical with the practical units.
2- Its units are same whether build up from the electromagnetic or electrostatic theory.
3. The rather cumbersome conversions necessary to relate the units of the cm. and es. C.G.S. systems to those of the practical system are avoided in this system.

In July 1950 the international electro-technical commission recommended the ampere as the fourth fundamental unit of rationalized M.K.S. system. This is the reason that this system is sometimes called as rationalized M.K.S.A. system.

7- SI Units

At the Eleven General conference on Weights and measures (La Conference General des Poids et Measures, CGPM) held in Paris in 1960 it was agreed to adopt the International System of Units (System Internationale d'Unites, abbreviated as SI). Previously. the different systems of units had been in use in various countries causing great inconvenience in international uses.

Physical Quantity	Name of Unit	Unit Symbol
Length	meter	m
Mass	kilogramme	kg
Time	second	s
Electric current	Ampere	A
Thermodynamic Temperature	Kelvin	K
Luminous Intensity	Candela	cd

Table (1) Basic SI units

Fundamentally, SI units is an absolute M. K. S. system of units with addition of three more Basic (fundamental) units, namely ampere (A) for current as fourth basic unit of electrical-system, kelvin (K) for temperature as fourth basic unit of thermodynamic system and candela (cd) for luminous intensity as fourth basic unit of illumination system. Thus this system has six basic units. In addition, there are three supplementary units, namely radian (rad) for plane angle, steradian (sr) for solid angle and mol (n) for quantity of substance. The basic SI units are shown in table (1).

Electrical Derived SI units			
Quantity	Name of SI unit	Unit abbreviation	Unit
Power	Watt	W	J/s
Charge	Coulomb	C	As
Electrical potential Potential difference e.m.f	Volt	V	W/A
Capacitance	Farad	F	C/V
Resistance	Ohm	Ω	V/A
Magnetic flux	Weber	Wb	Vs
Magnetic flux density	Tesla	T	Wb/m^2
Inductance	Henry	H	Wb/A
Electrical field strength	Volt per meter	-----	V/m
Magnetic field strength	Ampere per meter	-----	A/m

Prefixes and corresponding symbols used in SI units		
Unit multiplier	Prefix	Symbol
1 000 000 000 000 = 10^{12}	Tera	T
1 000 000 000 = 10^{9}	Giga	G
1 000 000 = 10^{6}	Mega	M
1000 = 10^{3}	kilo	k
100 = 10^{2}	hecta	h
10 = 10^{1}	deca	da
0.1	deci	d
0.01	centi	c
0.001	milli	m
0.000 001	micro	µ
0.000 000 001	nano	n
0.000 000 000 001	pico	p
0.000 000 000 000 001	demto	f
0.000 000 000 000 000 001	atto	a

Dimensional Equations of Electrical Quantities

Quantity	*Symbol*	*Equations from which the dimensions are derived*	*Dimensions*
Charge	Q, q	$Q=IT$	$[TI]$
E.M.F.	V, E, e	$V=\frac{\text{Work}}{Q}$	$[ML^2T^{-3}I^{-1}]$
Resistance	R	$R=\frac{V}{I}$	$[ML^2T^{-3}I^{-2}]$
Capacitance	C	$C=\frac{Q}{V}$	$[M^{-1}L^{-2}T^4I^2]$
Inductance	$\mathfrak{L}$	$e=\mathfrak{L}\frac{di}{dt}$	$[ML^2T^{-2}I^{-2}]$
Magnetic flux	ϕ	$e=N\frac{d\phi}{dt}$	$[ML^2T^{-2}I^{-1}]$
Flux density	B	$B=\frac{\text{Flux}}{\text{Area}}=\frac{\phi}{A}$	$[MT^{-2}I^{-1}]$
M.M.F.	AT	$AT=NI$	$[I]$
Magnetizing force	H	$H=\frac{\text{m.m.f.}}{\text{Length}}=\frac{AT}{L}$	$[L^{-1}I]$
Reluctance	S	$S=\frac{AT}{\phi}$	$[M^{-1}L^{-2}T^2I^2]$
Electric flux	ψ	$\psi=Q$	$[TI]$
Electric flux density	D	$D=\frac{\text{Electric flux}}{\text{Area}}=\frac{\psi}{A}$	$[L^{-2}TI]$
Electric flux strength	E	$E=\frac{dV}{dt}$	$[ML^2T^{-4}I^{-1}]$

Chapter 3
Classification of instruments

1- Definition of instruments

An instrument is a device in which we can determine the magnitude or value of the quantity to be measured. The measuring quantity can be voltage, current, power and energy etc. Generally instruments are classified in to two categories:

A- **Absolute Instrument**

B- **Secondary Instrument**

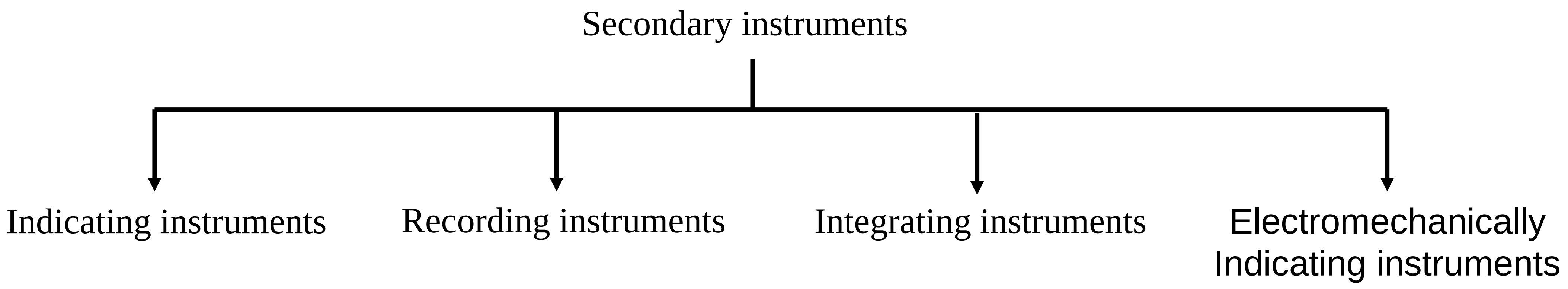
A- Absolute instrument

An absolute instrument determines the magnitude of the quantity to be measured in terms of the instrument parameter. This instrument is rarely used, because each time the value of the measuring quantities varies, we have to calculate the magnitude of the measuring quantity analytically, which is time consuming. These types of instruments are suitable for laboratory use. Example: Tangent galvanometer figure (6).

Figure (6) Tangent galvanometer

B- Secondary instrument

This instrument determines the value of the quantity to be measured directly. Generally these instruments are calibrated by comparing with another standard secondary instrument. Examples of such instruments are voltmeter, ammeter and wattmeter etc. Practically secondary instruments are suitable for measurement.



B.1- Indicating instrument

This instrument uses a dial and pointer to determine the value of measuring quantity, as shown in figure (7). The pointer indication gives the magnitude of measuring quantity.

B.2- Recording instrument

This type of instruments records the magnitude of the quantity to be measured continuously over a specified period of time. Figure (8) shows an example of recording instrument "UMG 503 Power Meter" used to record 3-phase voltages.

Figure (7) Indicating instrument

Figure (8) Recording instrument

B.3- Integrating instrument

This type of instrument gives the total amount of the quantity to be measured over a specified period of time. For example the energy meter as shown in figure (9).

Figure (9) energy meter

B.4- Electromechanical indicating instrument

For satisfactory operation electromechanical indicating instruments need three forces which are:

a- Deflecting force
b- Controlling force
c- Damping force

2- Deflecting force

When there is no input signal to the instrument, the pointer will be at its zero position. To deflect the pointer from its zero position, a force is necessary which is known as deflecting force. A system which produces the deflecting force is known as a deflecting system. Generally a deflecting system converts an electrical signal to a mechanical force.

2.1- Magnitude effect

When a current passes through the coil, Figure (10), it produces an imaginary bar of magnet. When a soft-iron piece is brought near this coil it get magnetized. Depending upon the current direction the poles are produced in such a way that there will be a force of attraction between the coil and the soft iron piece. This principle is used in moving iron attraction type instrument.

If two soft iron pieces are place near a current carrying coil there will be a force of repulsion between the two soft iron pieces. This principle is utilized in the moving iron repulsion type instrument.

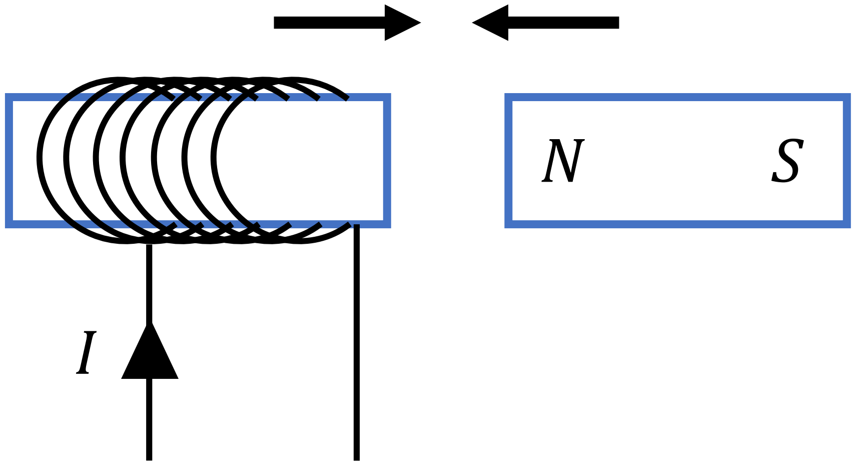


Figure (10) Magnitude effect

2.2- Force between a permanent magnet and a current carrying coil

When a current carrying coil is placed under the influence of magnetic field produced by a permanent magnet, a force is produced between them as shown in figure (11). This principle is utilized in the moving coil type instrument.

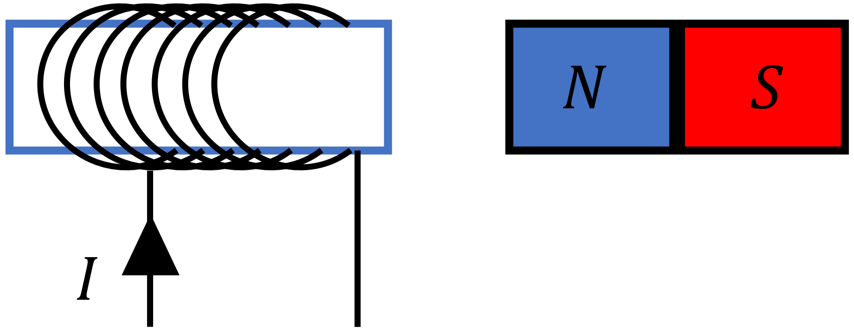


Figure (11) two magnetic fields

2.3- Force between two current carrying coil

When two current carrying coils are placed closer to each other there will be a force of repulsion between them. If one coil is movable and other is fixed, the movable coil will move away from the fixed one. This principle is utilized in electrodynamometer type instrument as shown in figure (12).

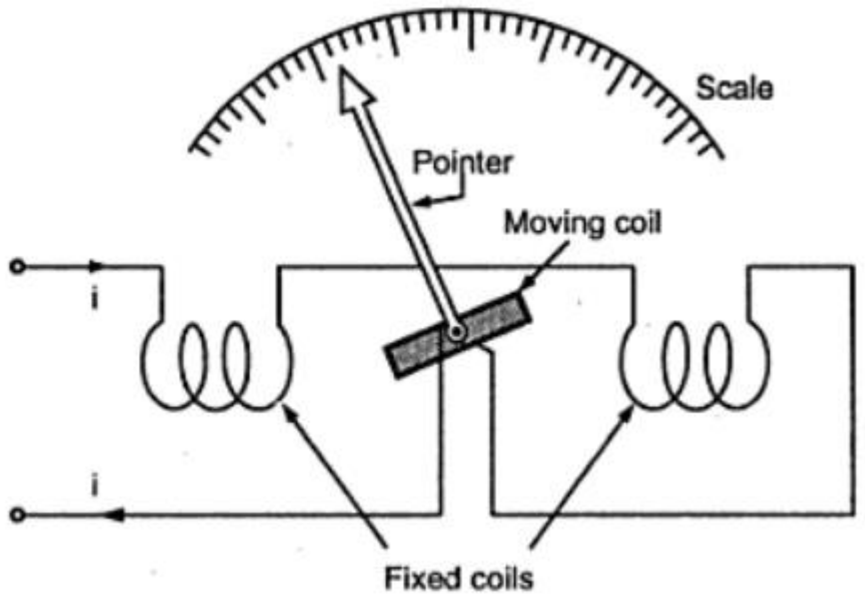


Figure (12) electrodynamometer

3- Controlling force

To make the measurement indicated by the pointer definite (constant) a force is necessary which will be acting in the opposite direction to the deflecting force. This force is known as controlling force. A system which produces this force is known as a controlled system. When the external signal to be measured by the instrument is removed, the pointer should return back to the zero position. This is possibly due to the controlling force and the pointer will be indicating a steady value when the deflecting torque is equal to controlling torque.

$T_d = T_c$

3.1- Spring control

Two springs are attached on either end of spindle figure (13). The spindle is placed in jewelled bearing, so that the frictional force between the pivot and spindle will be minimum. Two springs are provided in opposite direction to compensate the temperature error. The spring is made of phosphorous bronze. When a current is supply, the pointer deflects due to rotation of the spindle. While spindle is rotate, the spring attached with the spindle will oppose the movements of the pointer. The torque produced by the spring is directly proportional to the pointer deflection θ. $Tc \propto \theta$

The deflecting torque produced Td proportional to 'I'. When Tc = Td , the pointer will come to a steady position. Therefore $\theta \propto I$

Since, θ and I are directly proportional to the scale of such instrument which uses spring controlled is uniform.

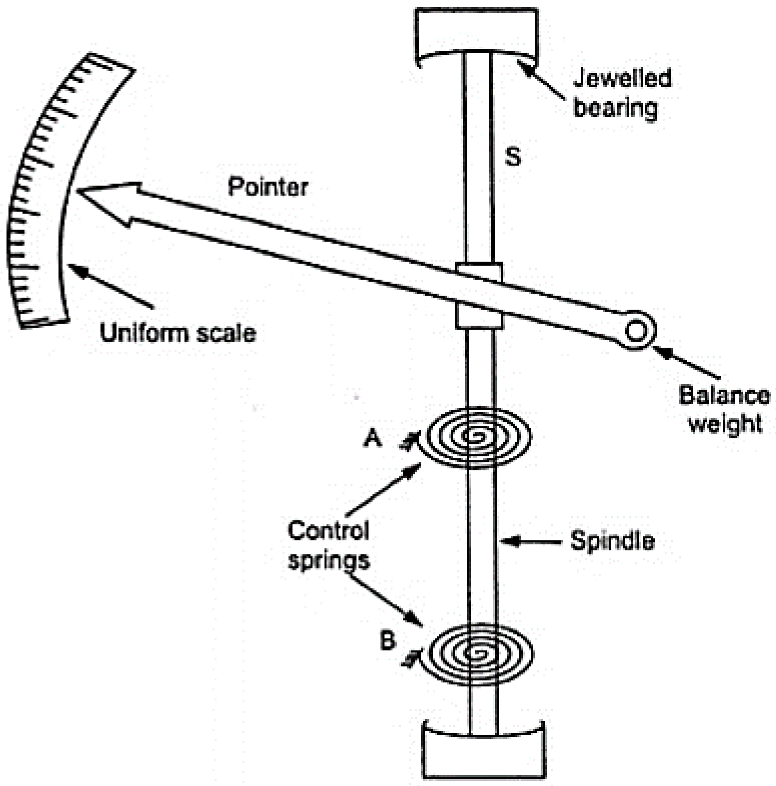


Figure (13) spring control

4- Damping force

Due to inertia produced by this system, the pointer oscillates about it final steady position before coming to rest. The time required to take the measurement is somehow higher than the other types. To damp out the oscillation quickly, a damping force is necessary. This force is produced by different systems:

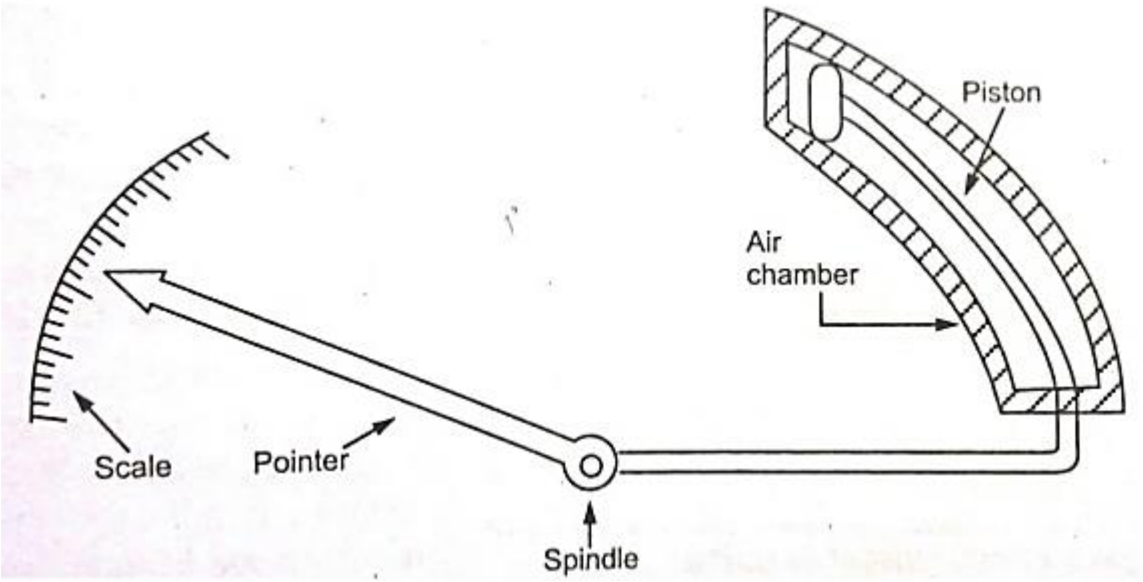


Figure (14) Air friction damping

4.1- Air friction damping
4.2- Eddy current damping
4.3- Fluid friction damping

4.1- Air friction damping

The piston is mechanically connected to a spindle through the connecting rod figure (14). The pointer is fixed to the spindle moves over a calibrated dial. When the pointer oscillates in clockwise direction, the piston goes inside and the cylinder gets compressed. The air pushes the piston upwards and the pointer tends to move in anticlockwise direction. If the pointer oscillates in anticlockwise direction the piston moves away and the pressure of the air inside cylinder gets reduced. The external pressure is more than that of the internal pressure. Therefore the piston moves down wards. The pointer tends to move in clock wise direction.

4.2- Eddy current damping

An aluminium circular disc is fixed to the spindle figure (15). This disc is made to move in the magnetic field produced by a permanent magnet. When the disc oscillates it cuts the magnetic flux produced by damping magnet. An e.m.f is induced in the circular disc by faradays law. Eddy currents are established in the disc since it has several closed paths. By Lenz's law, the current carrying disc produced a force in a direction opposite to oscillating force. The damping force can be varied by varying the projection of the magnet over the circular disc.

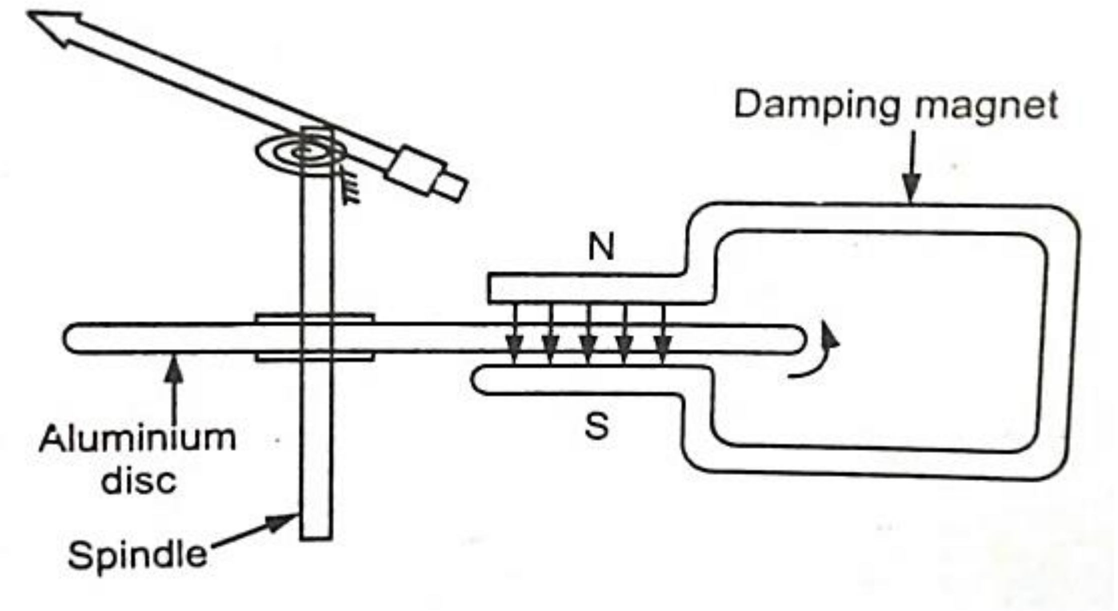


Figure (15) Eddy current damping

5- Permanent Magnet Moving Coil (PMMC) instrument

One of the most accurate type of instrument used for D.C. measurements is PMMC instrument.

5.1- Construction: A permanent magnet is used in this type instrument. Aluminium former is provided in the cylindrical in between two poles of the permanent magnet figure (16). Coils are wound on the aluminium former which is connected with the spindle. This spindle is supported with jewelled bearing. Two springs are attached on either ends of the spindle. The terminals of the moving coils are connected to the spring. Therefore the current flows through spring 1, moving coil and spring 2.

Damping: Eddy current damping is used. This is produced by aluminium former.

Control: Spring control is used.

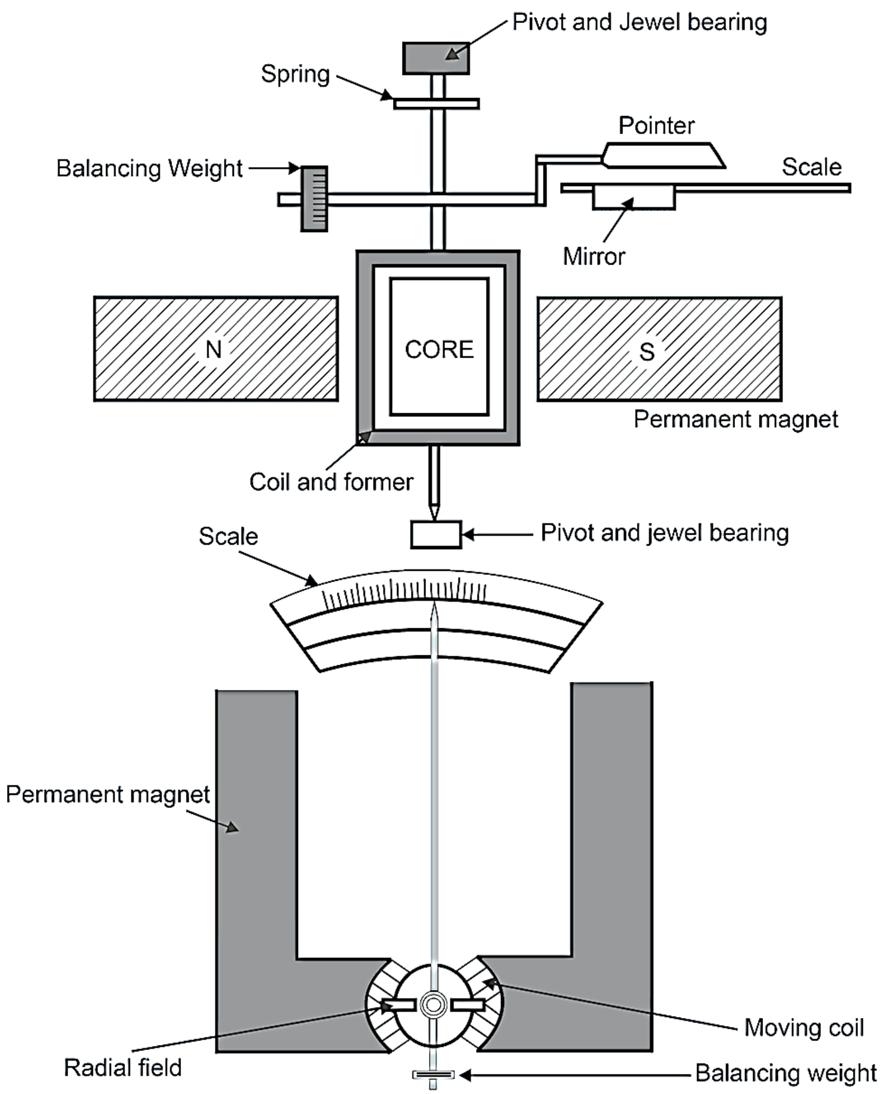


Figure (16) PMMC

5.2- Principle of operation: When D.C. supply is given to the moving coil, D.C. current flows through it. When the current carrying coil is kept in the magnetic field, it experiences a force. This force produces a torque and the former rotates. The pointer is attached with the spindle. When the former rotates, the pointer moves over the calibrated scale. When the polarity is reversed a torque is produced in the opposite direction. The mechanical stopper does not allow the deflection in the opposite direction. Therefore the polarity should be maintained with PMMC instrument. If A.C. is supplied, a reversing torque is produced. This cannot produce a continuous deflection. Therefore this instrument cannot be used in A.C.

5.3- Torque developed by PMMC

$Td = deflecting\ torque$

$TC = controlling\ torque$

$\theta = angle\ of\ deflection$

$K = spring\ constant$

$b = width\ of\ the\ coil$

$l = height\ of\ the\ coil\ or\ length\ of\ coil$

$N = No.of\ turns$

$I = current$

$B = Flux\ density$

$A = area\ of\ the\ coil$

5.4- The force produced in the coil is given by

$F = BIL\sin\theta$

$When\ \theta = 90°$

$For\ N\ turns, F = NBIL$

$Torque\ produced\ Td = F \times \perp r\ distance$

$Td = NBIL * b = BINA$

$Td = BANI$

$Td \propto I$

5.5- Advantages & Disadvantages

Advantages

- Torque/weight is high
- Power consumption is less
- Scale is uniform
- Damping is very effective
- Since operating field is very strong, the effect of stray field is negligible
- Range of instrument can be extended

Disadvantages

- Use only for D.C.
- Cost is high
- Error is produced due to ageing effect of PMMC
- Friction and temperature error are present

6- Moving Iron (MI) instruments

One of the most accurate instrument used for both AC and DC measurement is moving iron instrument. There are two types of moving iron instrument:

- Attraction type
- Repulsion type

6.1 Attraction type M.I. instrument

6.1.1- Construction: The moving iron fixed to the spindle is kept near the hollow fixed coil figure (17). The pointer and balance weight are attached to the spindle, which is supported with jewelled bearing. Here air friction damping is used.

6.1.2- Principle of operation

The current to be measured is passed through the fixed coil. As the current is flow through the fixed coil, a magnetic field is produced. By magnetic induction the moving iron gets magnetized. The north pole of moving coil is attracted by the south pole of fixed coil. Thus the deflecting force is produced due to force of attraction. Since the moving iron is attached with the spindle, the spindle rotates and the pointer moves over the calibrated scale. But the force of attraction depends on the current flowing through the coil.

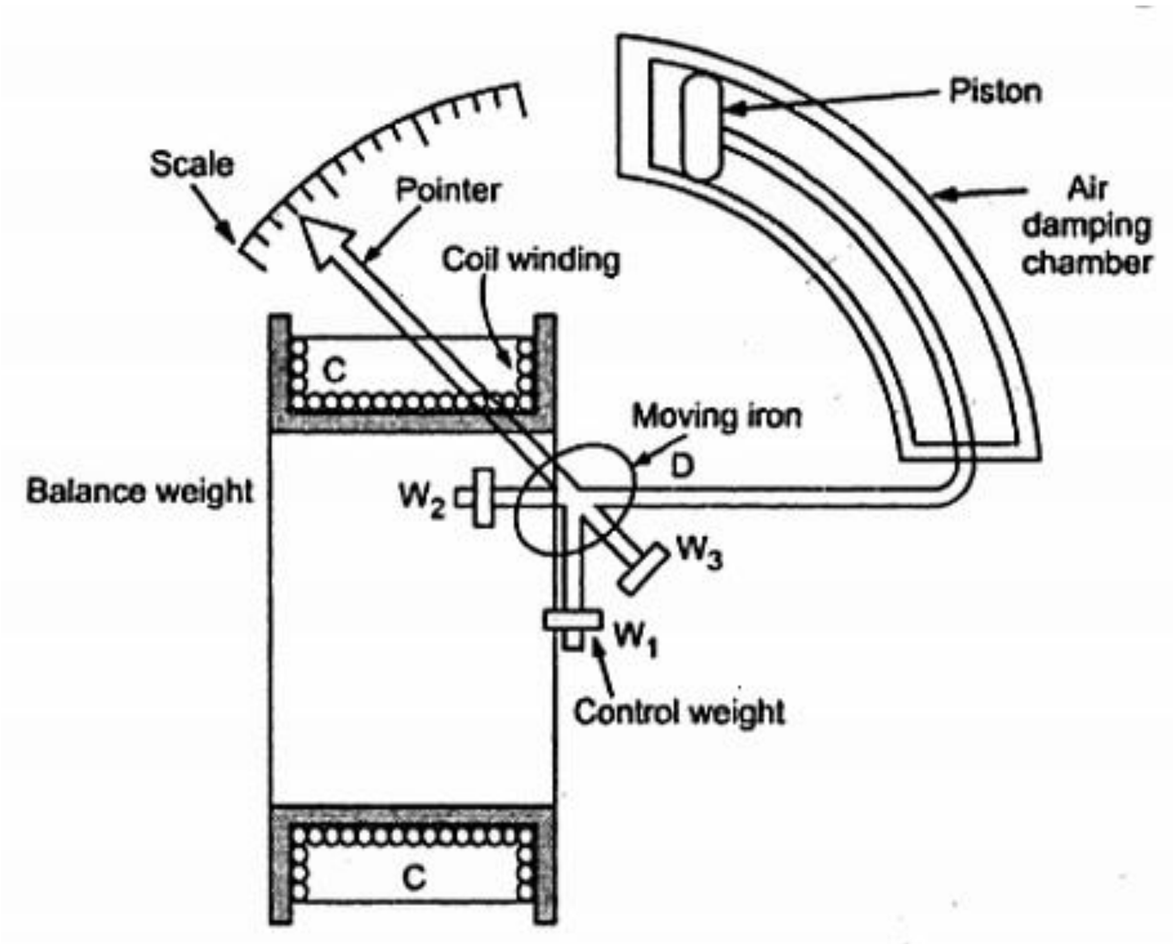
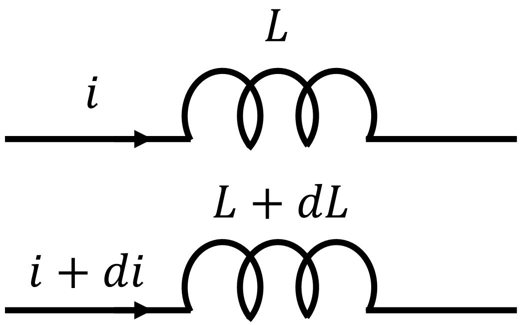


Figure (17) Attraction type M.I. instrument

6.1.3- Torque developed by M.I

Let ‘θ ’ be the deflection corresponding to a current of ‘i’ amp

Let the current increases by di, the corresponding deflection is ‘θ + dθ ’



There is change in inductance since the position of moving iron change w.r.t the fixed electromagnets.

Let the new inductance value be ‘L + dL’. The current change by ‘di’ is dt seconds.

Let the emf induced in the coil be ‘e’ volt.

$$e = \frac{d}{dt}(Li) = L\frac{di}{dt} + i\frac{dL}{dt}$$

Multiply by idt

$$e = \frac{d}{dt}(Li) * (idt) = L\frac{di}{dt} * (idt) + i\frac{dL}{dt} * (idt) \Rightarrow e\,idt = Lidi + i^2dL$$

This equation gives the energy is used in to two forms. Part of energy is stored in the inductance. Remaining energy is converted in to mechanical energy which produces deflection.

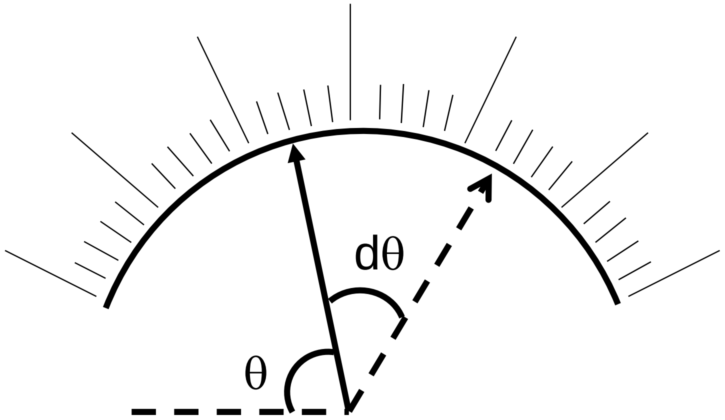


Figure (18)

Change in energy stored = (Final energy - initial energy stored)

$$= \frac{1}{2}(L + dL)(i + di)^2 - \frac{1}{2}Li^2$$
$$= \frac{1}{2}\{(L + dL)(i^2 + di^2 + 2idi) - Li^2\}$$
$$= \frac{1}{2}\{(L + dL)(i^2 + 2idi) - Li^2\}$$
$$= \frac{1}{2}\{Li^2 + 2Lidi + i^2dL + 2ididL - Li^2\}$$
$$= \frac{1}{2}\{2Lidi + i^2dL\}$$
$$= Lidi + \frac{1}{2}i^2dL$$

Mechanical work to move the pointer by $d\theta$

$$= T_d d\theta$$

By law of conservation of energy,

Electrical energy supplied = (Increase in stored energy + mechanical work done)

Input energy = Energy stored + Mechanical energy

$$Li\,di + i^2dL = Lidi + \frac{1}{2}i^2dL + T_d d\theta$$
$$\frac{1}{2}i^2dL = T_d d\theta \Longrightarrow T_d = \frac{1}{2}i^2\frac{dL}{d\theta}$$

At steady state condition Td = TC

$$\frac{1}{2}i^2\frac{dL}{d\theta} = T_d = K\theta \Longrightarrow \theta = \frac{1}{2K}i^2\frac{dL}{d\theta}$$

$\theta \propto i^2$

When the instruments measure AC, $\theta \propto {i^2}_{rms}$

Scale of the instrument is non uniform

Advantages

- MI can be used in AC and DC
- It is cheap
- Supply is given to a fixed coil, not in moving coil.
- Simple construction
- Less friction error.

Disadvantages

- It suffers from eddy current and hysteresis error
- Scale is not uniform
- It consumed more power
- Calibration is different for AC and DC operation

6.2- Repulsion type moving iron instrument

6.2.1- Construction: The repulsion type instrument has a hollow fixed iron attached to it figure (19). The moving iron is connected to the spindle. The pointer is also attached to the spindle in supported with jewelled bearing.

6.2.2- Principle of operation: When the current flows through the coil, a magnetic field is produced by it. So both fixed iron and moving iron are magnetized with the same polarity, since they are kept in the same magnetic field. Similar poles of fixed and moving iron get repelled. Thus the deflecting torque is produced due to magnetic repulsion. Since moving iron is attached to spindle, the spindle will move. So that pointer moves over the calibrated scale.
Damping: Air friction damping is used to reduce the oscillation.
Control: Spring control is used.

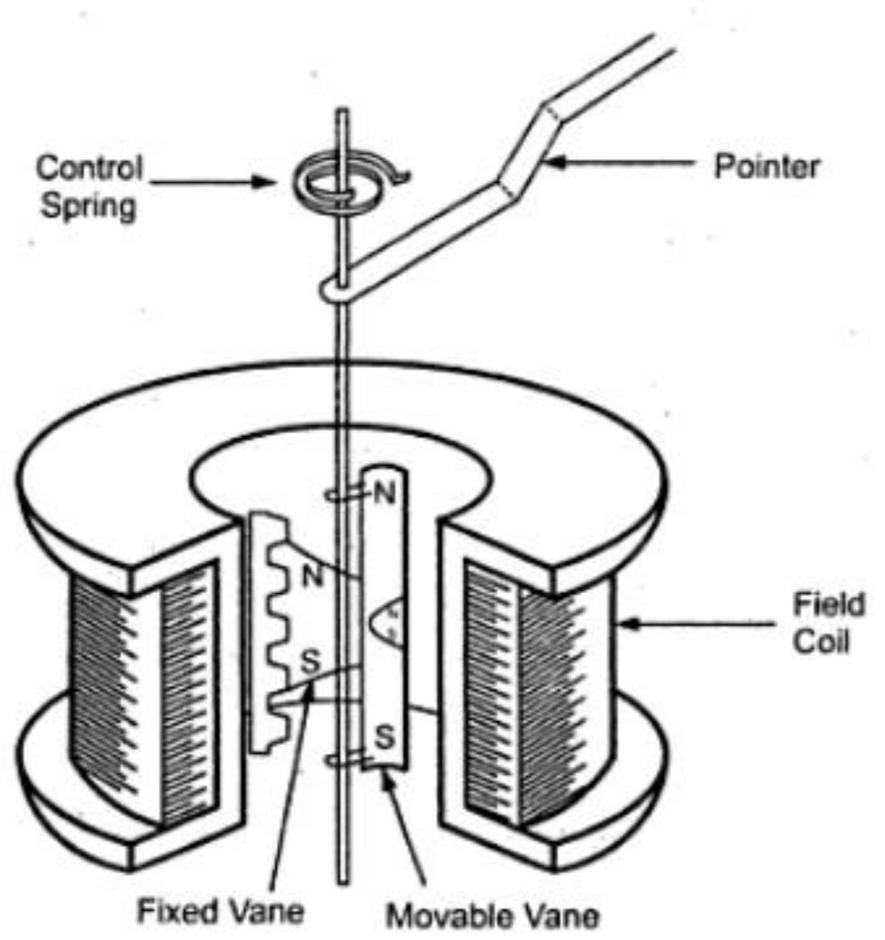


Figure (19) Repulsion type moving iron instrument

7- Characteristics of meter

7.1- Full scale deflection current(IFSD)

The current required to bring the pointer to full-scale or extreme right side of the instrument is called full scale deflection current. It must be as small as possible. Typical value is between 2μ A to 30mA.

7.2- Resistance of the coil(*Rm*)

This is ohmic resistance of the moving coil. It is due to $\rho,\ \ l$ and A. For an ammeter this should be as small as possible.

7.3- Sensitivity of the meter(S)

$$S=\frac{1}{I_{FSD}}\left(\frac{\Omega}{volt}\right), \uparrow S=\frac{Z\uparrow}{V}$$

It is also called ohms/volt rating of the instrument. Larger the sensitivity of an instrument, more accurate is the instrument. It is measured in Ω/volt. When the sensitivity is high, the impedance of meter is high. Hence it draws less current and loading affect is negligible. It is also defend as one over full scale deflection current.

8-Error in moving iron (M.I) instrument

8.1- Temperature error

Due to temperature variation, the resistance of the coil varies. This affects the deflection of the instrument. The coil should be made of manganic, so that the resistance is almost constant.

8.2- Hysteresis error

Due to hysteresis affect the reading of the instrument will not be accurate. When the current is decreasing, the flux produced will not decrease instantaneously. Due to this the meter reads a higher value of current. Similarly when the current increases the meter reads a lower value of current. This produces error in deflection. This error can be eliminated using small iron parts with narrow hysteresis loop so that the demagnetization takes place very quickly.

8.3- Eddy current error

The eddy currents induced in the moving iron affect the deflection. This error can be reduced by increasing the resistance of the iron.

8.4- Stray field error

Since the operating field is weak, the effect of stray field is high. Due to this, error is produced in deflection. This can be eliminated by shielding the parts of the instrument.

8.5- Frequency error

When the frequency changes the reactance of the coil changes.

Deflection of moving iron voltmeter depends upon the current through the coil. Therefore, deflection for a given voltage will be less at higher frequency than at low frequency. A capacitor is connected in parallel with multiplier resistance. The net reactance, (X L − XC) is very small, when compared to the series resistance. Thus the circuit impedance is made independent of frequency. This is because of the circuit is almost resistive.

$$Z = \sqrt{(Rm + Rs)^2 + (XL)^2}$$

$$I = \frac{V}{Z} \Longrightarrow I = \frac{V}{\sqrt{(Rm + Rs)^2 + (XL)^2}}$$

$$C = 0.41\frac{L}{Rs^2}$$

9- Multi range Ammeter

When the switch is connected to position (1) as shown in figure (20), the supplied current I1

$$I_{sh1}R_{sh1} = I_m R_m \Longrightarrow R_{sh1} = \frac{I_m R_m}{I_{sh1}}$$

$$\because I_1 = I_{sh1} + I_m , \qquad \therefore R_{sh1} = \frac{I_m R_m}{I_1 - I_m}$$

$$R_{sh1} = \frac{R_m}{\frac{I_1}{I_m} - 1} \Longrightarrow R_{sh1} = \frac{R_m}{m1 - 1} \qquad m1 = \frac{I_1}{I_m}$$

$$m = Multiplying\ power\ of\ shunt$$

$$R_{sh2} = \frac{R_m}{m2 - 1} \qquad m2 = \frac{I_2}{I_m}$$

$$R_{sh3} = \frac{R_m}{m3 - 1} \qquad m3 = \frac{I_3}{I_m}$$

$$R_{sh4} = \frac{R_m}{m4 - 1} \qquad m4 = \frac{I_4}{I_m}$$

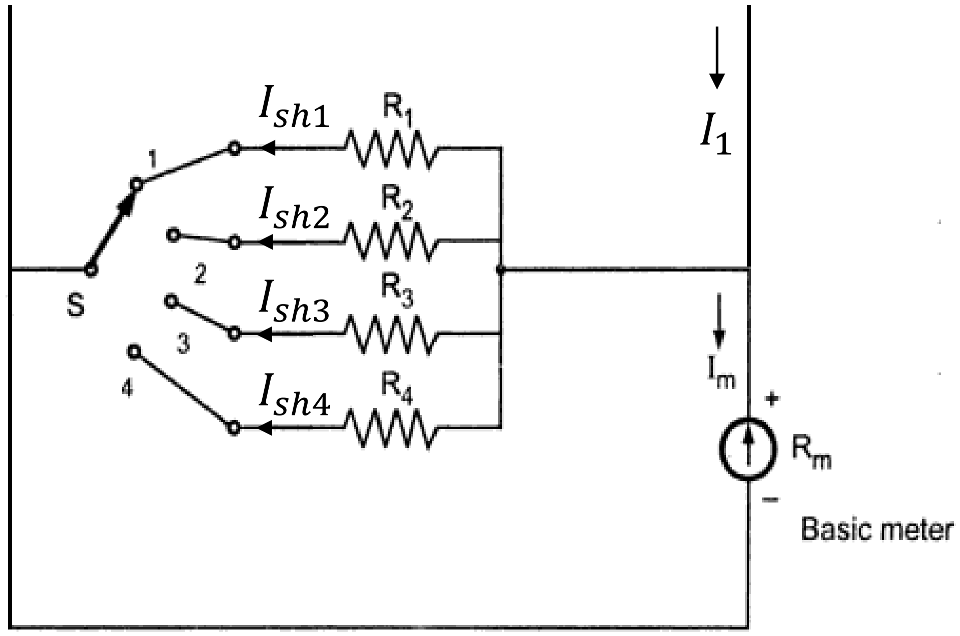


Figure (20) Multi range Ammeter circuit

10- Ayrton shunt

Ayrton shunt is also called universal shunt as shown in figure (21). Ayrton shunt has more sections of resistance. Taps are brought out from various points of the resistor. The variable points in the o/p can be connected to any position. Various meters require different types of shunts. The Ayrton shunt is used in the lab, so that any value of resistance between minimum and maximum specified can be used. It eliminates the possibility of having the meter in the circuit without a shunt.

R1 = Rsh1 – Rsh2
R2 = Rsh2 – Rsh3
R3 = Rsh3 – Rsh4
R4 = Rsh4

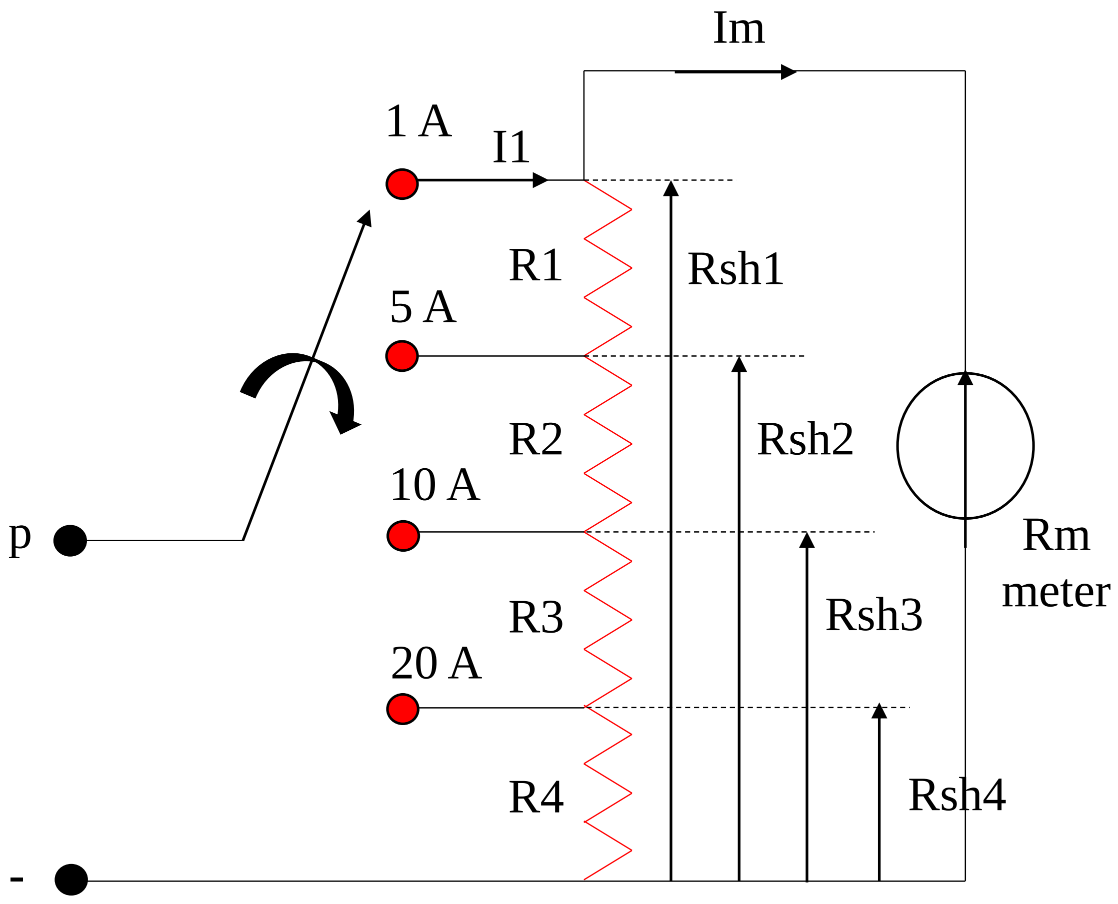


Figure (21) Ayrton shunt circuit

11- Multi range D.C. voltmeter

We can obtain different Voltage ranges by connecting different value of multiplier resistor in series with the meter as shown in figure (22). The number of these resistors is equal to the number of ranges required.

$$Rs1 = Rm\,(m1 - 1) \qquad m1 = \frac{V1}{Vm}$$
$$Rs2 = Rm\,(m2 - 1) \qquad m2 = \frac{V2}{Vm}$$
$$Rs3 = Rm\,(m3 - 1) \qquad m3 = \frac{V3}{Vm}$$

Example 3.1
Example 3.2
Example 3.3
Example 3.4
Example 3.5
Example 3.6
Example 3.7

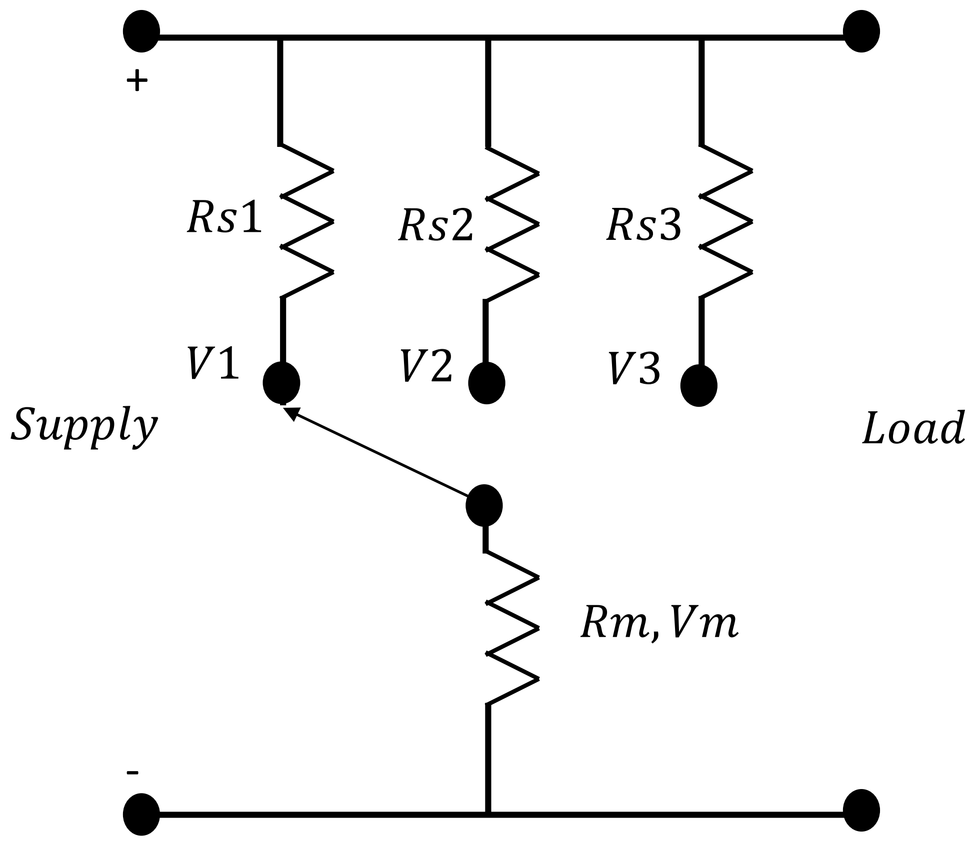


Figure (22) Multi range D.C. voltmeter

Chapter 4
Various measurements methods for determining resistance, inductance, and capacitance

1- Various measurements methods for determining resistance

The choice of a Suitable method of measurement of resistances depends on so many factors. The value of resistance to be measured is the most important factor. Other factors may be the accuracy of measurements, working conditions etc. The second point is important as we know that the resistance of conducting materials increases "with the temperature, while that of insulating materials decreases with the temperature rise and the absorption of moisture.
The classification of resistances, from the point of View of measurements, is done as follows :

Low resistance: All resistances of the order of 1Ω and lower values come under this class of resistances. The examples of such resistances are : (a) armature and series winding resistances of large machines **;** (b) resistances of ammeter shunts.
Medium resistances: This class includes all resistances of values between about 1Ω and 100 kΩ, Most of the electrical apparatus used in practice have resistances within this limit.
High Resistances: Resistances of 100 KΩ and above may be termed as high resistances. The examples may be the insulation resistance of cables, resistances of dielectrics etc.

1.1- Measurement of Low resistance:

The necessity bf considering the measurement of low resistances separately from that medium resistances has arisen from the fact that contact resistances of the order of, say, 0.001 Ω through negligible in comparison with medium resistances, are of great importance for low resistances such as of 0.01 Ω. Further, in measurement of low resistances the potential points across which the volt drop is to be taken for comparison must be accurately defined.

The direct reading instrument for the measurement of low resistances is the Ductor ohmmeter. The following laboratory methods of measuring low resistances are common:

1.1.1- Ammeter and voltmeter method
1.1.2- Potentiometer method
1.1.3- Kelvin double bridge method

Ductor ohmmeter

1.1.1- Ammeter and voltmeter method

Ammeter and Voltmeter Method. This method of measuring resistances is the simplest of all methods and is commonly used for the measurement of low resistances, if the accuracy required is of the order of l per cent. The main reasons, in addition to those common in all methods, of this method being rough and the accuracy being poor are:

1.The shunting effect of the voltmeter
2.The error in ammeter indication .
3.The error in voltmeter indication.
The circuit diagram of this method is shown in Figure (23).

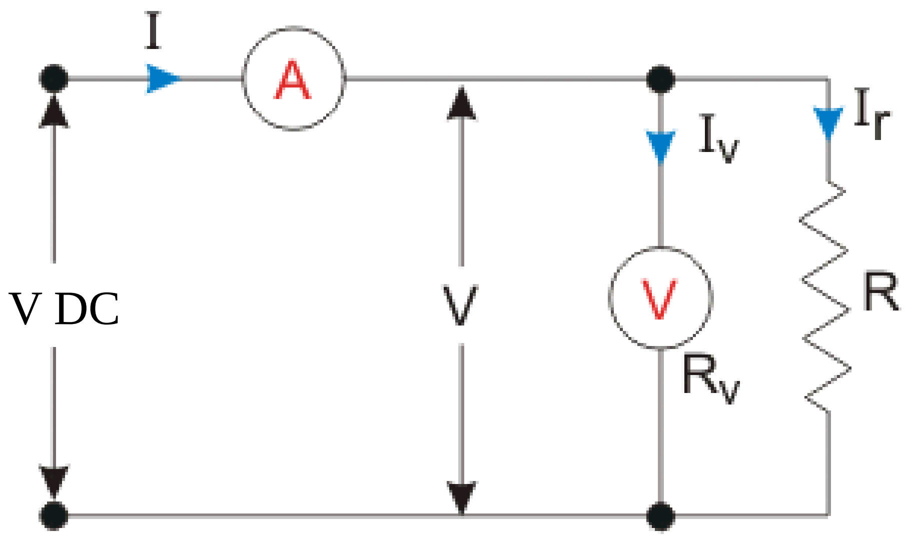


Figure (23) Ammeter and voltmeter method

R is unknown resistance to be measured.
V is a high resistance voltmeter.
Rv is the resistance of the voltmeter.
A is ammeter connected in series with a regulating rheostat Rh.
An example of regulating rheostat is shown in figure (24)

$$R = \frac{Voltmeter\ reading}{Ammeter\ reading} = \frac{V}{I}$$

Example 4.1

This example has shown that this method can be used for measurement of low resistances of high current currying capacity because in that case only the volt-drop across the resistance will be high and a high range, high resistance voltmeter can be used to minimize the error due to shunting effect of the voltmeter. In case this method is used for the measurement of low resistances of low current carrying capacity, it is essential to make correction for the shunting effect of the voltmeter.

If Rm is the measured value of the resistance then:

The current through the unknown resistance R is:

$$I' = \frac{R_v}{R_v + R} I \qquad V = \frac{RR_v}{R + R_v} I \qquad R_m = \frac{V}{I} = \frac{RR_v}{R + R_v}$$

where I is the ammeter reading, Rv the resistance of the voltmeter, and V is the voltage of the voltmeter.

$$Rm = \frac{RR_v}{R + R_v} \Longrightarrow R = \frac{R_m R_v}{R_v - R_m} = KR_m$$

Where correction factor $K = \frac{R_v}{R_v - R_m}$

Figure (24) regulating rheostat

1.1.2- Potentiometer method

A potentiometer is, essentially, an instrument by which two e.m.f.'s (or potential differences) are compared. If one of them is known then the other can be known by comparison with the former. A potentiometer can be used to measure unknown resistance. The value of the unknown resistance is obtained by comparison with a standard resistance. The standard resistance is specified at a certain temperature. Thus, it is essential to measure the temperature of both the resistances. The value of the standard resistance should therefore be obtained at the measured temperature and the unknown resistance is calculated corresponding to that value of the standard resistance. Thus, the resistance of the unknown resistor is obtained at the measured temperature and, therefore, the temperature must also be Specified with the resistance.

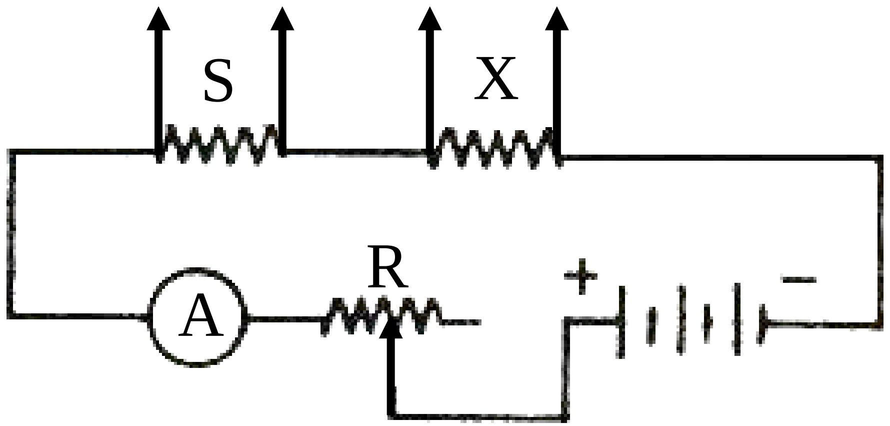


Figure (25) Potentiometer method of measuring resistance

The way of measurement is as follow:
A known current is passed through the resistor and the volt-drop across it is measured by comparison. The dc potentiometer may be used for measuring resistances of very low value, such as instrument shunts. This is really a laboratory or test room method, and is based on the comparison of one resistance against another by an indirect method. The voltage drop across both the unknown resistor and standard resistor R are measured by a dc potentiometer as shown in figure (25). The ratio of two potentiometer readings gives the ratio of X to S Mathematically.

1.1.3- Kelvin double bridge method

The Kelvin double bridge is one of the best available bridge for the precise measurement of low resistance. The provision, in this bridge has been made to eliminate the errors due to contact and lead resistance. The connections are shown in Figure (26) In the figure, X and S are unknown and standard resistances respectively. r is a low resistance like connecting adjacent current terminals of X and S. Q, M, q and m are four non-inductive resistances, one pair of which (Q and q or M and m) are variable. G is a dc. galvanometer used as a null detector. The balance may be obtained by varying the ratio Q/M. but in practice the ratio Q/M (=q/m) is variable in steps and kept fixed at a value roughly equal to X/S.
Finally, the balanced is obtained by varying the value of S.

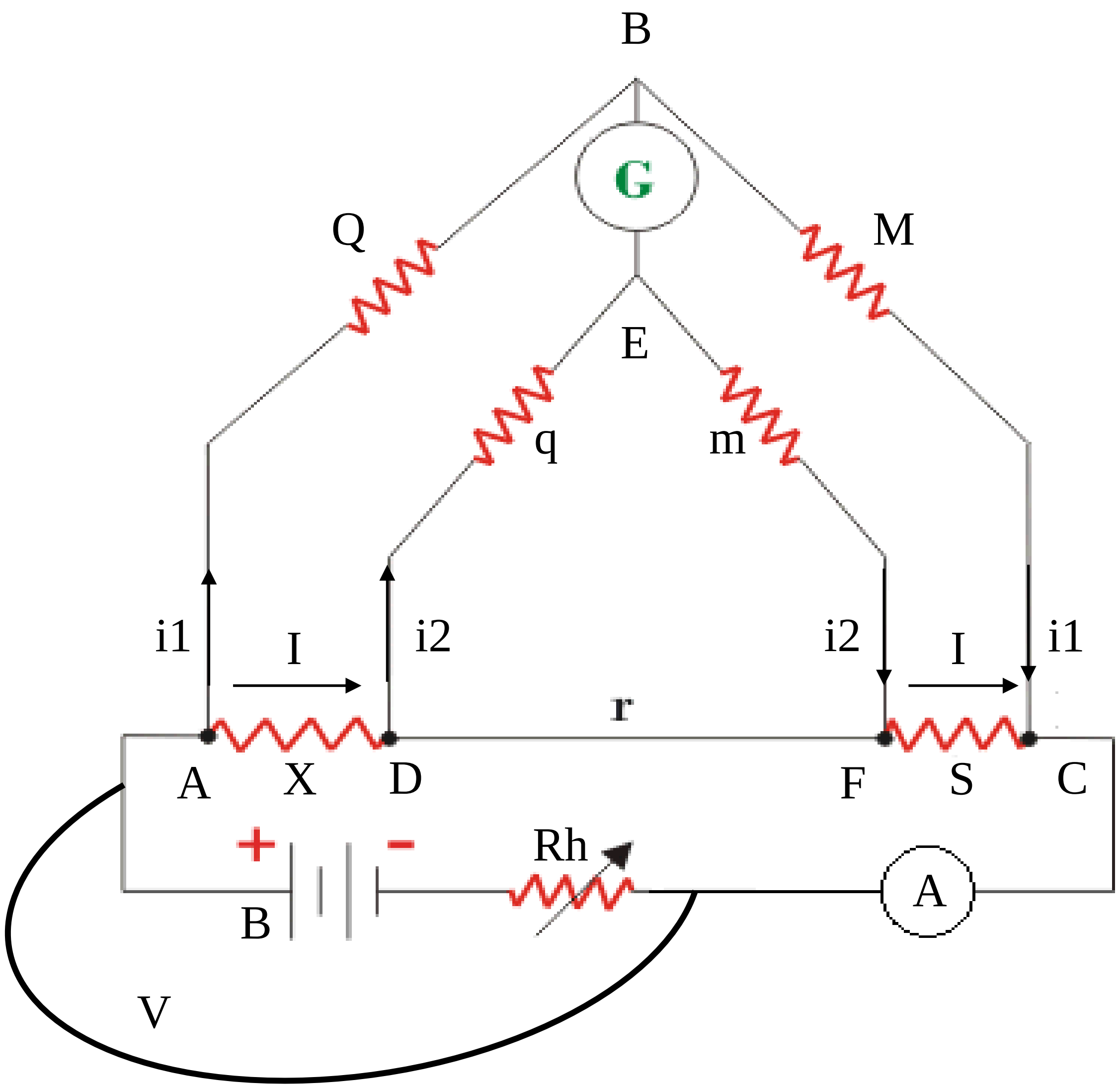


Figure (26) Kelvin double bridge method

For this, S is made of a fixed standard resistance S' and a variable shunt s. The shunt is usually a resistance box.

Therefore, $\frac{1}{S} = \frac{1}{S'} + \frac{1}{s} \Rightarrow S = \frac{S's}{S'+s}$

Sometimes instead of varying S , the unknown resistance X is shunted and the balance is obtained by varying the shunt resistance. The measured value of X is given by $\frac{X}{S} = \frac{Q}{M} = \frac{q}{m}$

Theory: After $\Delta - Y$ transforomation of q, m and r and representing the battery and rheostat by simply a battery of e.m.f. V. The figure (26) can be redrawn as in Figure (27).

Where, $x' = \frac{rq}{r+q+m}$; $s' = \frac{rm}{r+q+m}$; $r' = \frac{qm}{r+q+m}$

When the bridge is balanced the current through the galvanometer is zero and the current distribution is, therefore, as shown in figure (27). Also

$i1Q = i2(X + x') \quad and \quad i1M = i2(S + s')$

$$\frac{Q}{M} = \frac{X + x'}{S + s'}$$

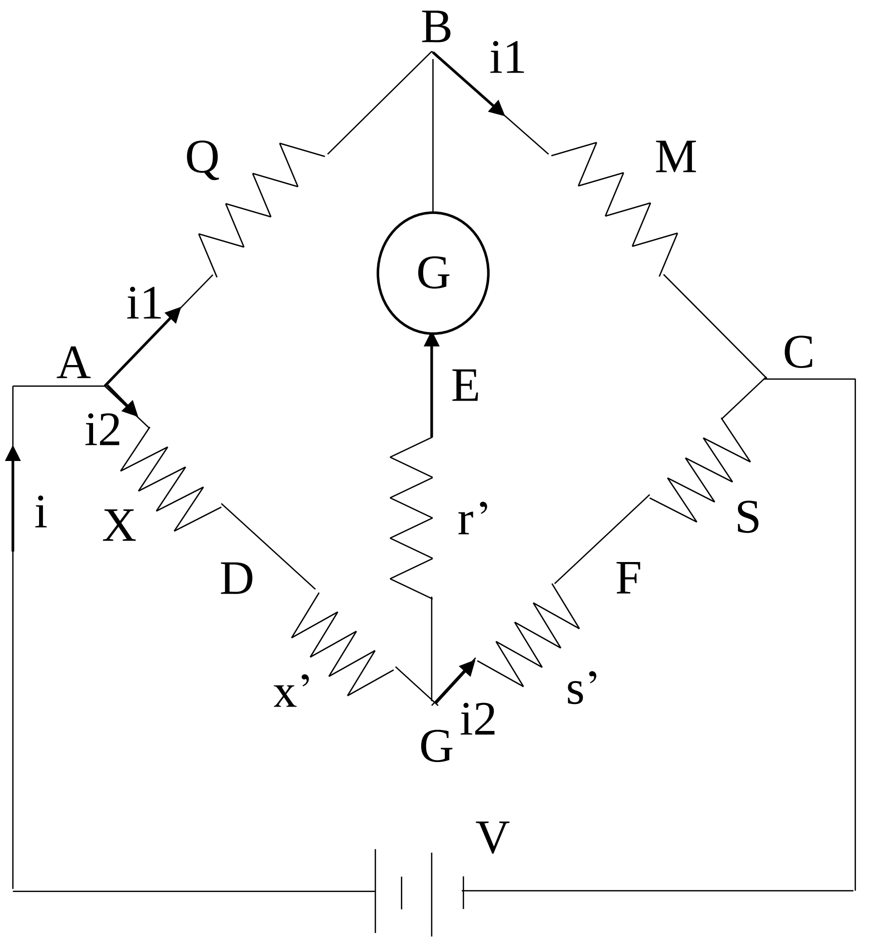


Figure (27) equivalent circuit of Kelvin double bridge

$i1Q = i2(X + x') \quad and \quad i1M = i2(S + s')$

$$\frac{Q}{M} = \frac{X + x'}{S + s'}$$

$$X = \frac{Q}{M}S + \frac{Q}{M}s' - x'$$

After substituting values of s' and x'

$$X = \frac{Q}{M}S + \frac{Q}{M}\frac{rm}{r + q + m} - \frac{rq}{r + q + m} \implies X = \frac{Q}{M}S + \frac{rm}{r + q + m}\left(\frac{Q}{M} - \frac{q}{m}\right)$$

The term $\frac{rm}{r+q+m}\left(\frac{Q}{M} - \frac{q}{m}\right)$ can be made very small by making r very small and also by making Q/M as nearly equal to q/m as possible.

Hence, $X = \frac{Q}{M}S \quad but \ \frac{Q}{M} = \frac{q}{m} \qquad \therefore \ \frac{X}{S} = \frac{Q}{M} = \frac{q}{m}$

To eliminate the error due to thermoelectric e.m.f.'s, a measurement with the direction of the current reversed should also be taken and the mean of the two measured values should be taken as the correct value of X.

1.2- DC Bridge circuit

Bridge have long been used for the measurement of resistance R, inductance L and capacitance C while various AC bridges are used for the L and C measurement, one DC bridge is traditionally used for the measurement of R.

1.2.1- Wheatstone Bridge

A Wheatstone bridge, shown in figure (28), is an electrical circuit developed by Charles Wheatstone, and it is used to determine the value of an unknown electrical resistance in the circuit. Wheatstone bridge is highly capable in calculating very low valued resistances which other instruments like multimeter does not calculate accurately.

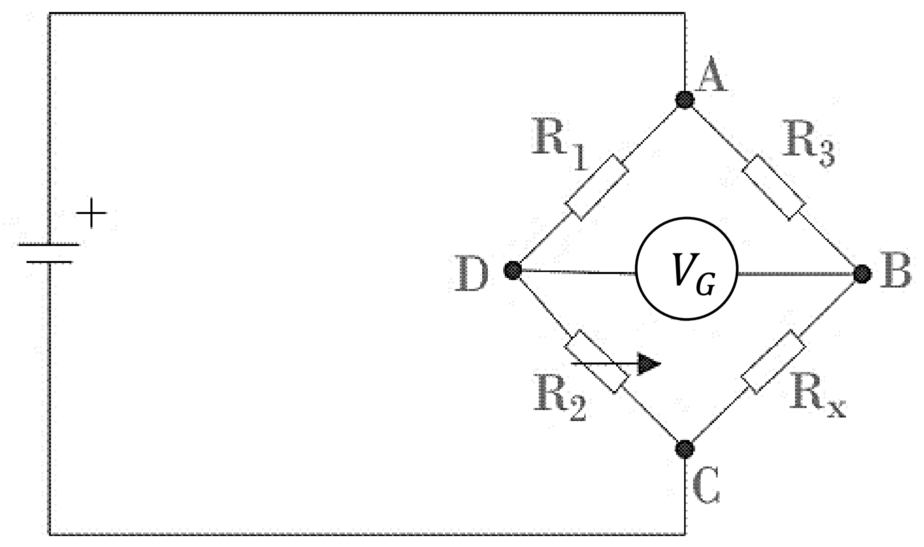


Figure (28) Wheatstone bridge

It has two parallel legs and each leg having two resistors in series. A third leg connected between the two parallel legs at some point within the legs, as drawn in figure. Among the four resistors, one resistance value can be determined by balancing the two legs. Out of four resistors, the value of two resistors R_1 and R_3 are known, the value of R_2 is adjustable, and the value of R_x is to be calculated. Then this adjustment is connected to electric supply and a galvanometer between terminal D and terminal B. Now the value of an adjustable resistor is adjusted until the ratio of the two branches resistances become equal i.e. $(R_1/R_2) = (R_3/R_x)$, and galvanometer reads zero as current stop flowing through the circuit. Now the circuit is balanced and the value of the unknown resistor could be measured easily. The reading of the R_3 decides the direction of the flow of current.

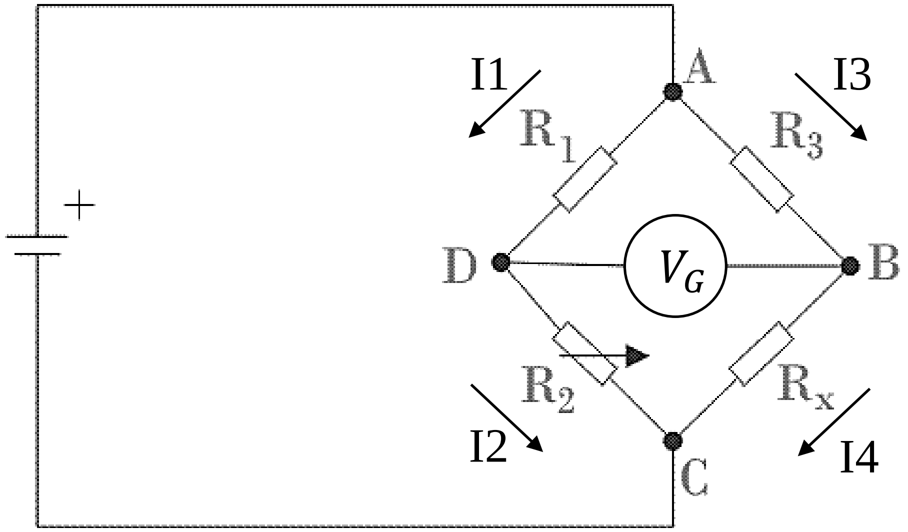


Figure (28) Wheatstone bridge

A bridge is balanced when the voltage at point D equals the voltage at point B. $I1R1 = I3R3 \quad and \quad I2R2 = I4R4$

$$I1 = I2 \quad and \quad I3 = I4 \quad \therefore \quad \frac{I1R1}{I2R2} = \frac{I3R3}{I4R4} \Longrightarrow R4 = R_x = \frac{R_3R_2}{R_1}$$

To calculate the IG (galvanometer current), Rth and VG have to be calculated first with a given RG (galvanometer resistor).

$$VG = VD - VB = I1R1 - I3R3$$

$$Rth = \frac{R1R2}{R1 + R2} + \frac{R3RX}{R3 + Rx}$$

$$IG = \frac{VG}{Rth + RG}$$

Example 4.2

Homework 4.1

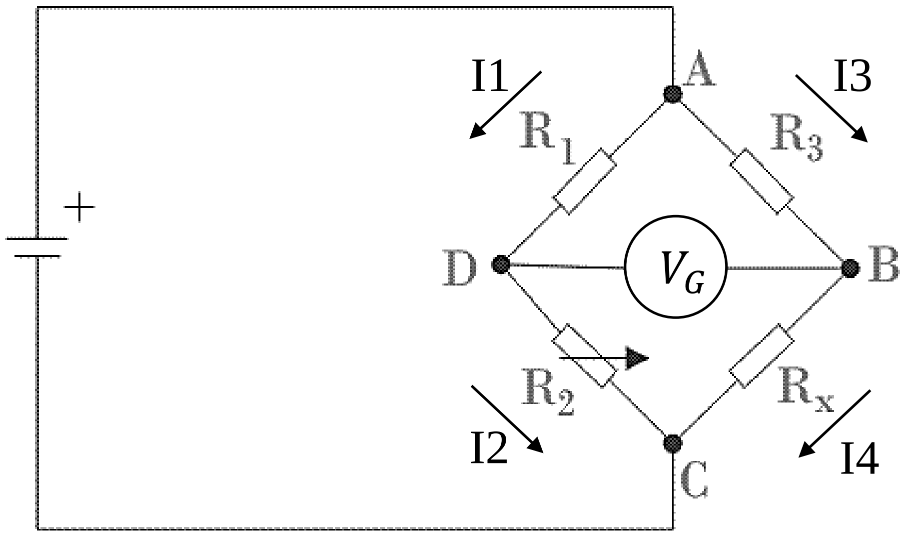


Figure (28) Wheatstone bridge

2- Various measurements methods for determining inductance and capacitance

Before descriibing a.c bridge methods of the measurement of inductance and capacitance, some of the approximate, but easily performable methods of thier measurements will be described here.

2.1- Ammeter and voltmeter methods

Like the measurement of resistance, the simplest method of measuring inductances may be the ammeter and voltmeter method. An alternating current of suitable value at power frequency is passed through the coil, the self-inductance of which is to be measured. The current is measured by an a.c. ammeter while the volt-drop across the coil is measured by a high resistance voltmeter. Then the impedance of the coil is.

$$Z = \frac{Voltmeter\ reading}{Ammeter\ reading} = \frac{V}{I}$$

But $\quad Z = \sqrt{R^2 + (2\pi f)^2 L^2}$

Or $\quad L = \frac{1}{2\pi f}\sqrt{Z^2 - R^2}$

Where R is the resistance of the coil and f is the frequency of current.
The a.c. resistance R may be taken same as the d.c. resistance, for the measurement of which various methods have already been described, to a close approximation if the current frequency is low.

To avoid the measurement of d.c. resistance an a.c. potentiometer is used to measure the current and the voltage-drop across the coil. A non-inductive resistance S is connected in series with the coil under test and the voltage drop across this, as well as that across the coil, is measured. If the voltage drop across the non inductive resistance S is Vs then the current.

$$I = \frac{V_S}{S}$$

After substituting this value of current in equation $Z = \sqrt{R^2 + (2\pi f)^2 L^2}$ and $Z = \frac{V}{I}$

$$\frac{S\,V}{V_S} = \sqrt{R^2 + (2\pi f)^2 L^2}$$

From the vector diagram shown in figure (29).

$$I * 2\pi f\ L = V \sin\theta$$

$$L = \frac{V}{2\pi f\ I} \sin\theta = \frac{S\ V \sin\theta}{V_S * 2\pi f}$$

And $IR = V\cos\theta$

$$R = \frac{V}{I}\cos\theta = \frac{SV}{V_S}\cos\theta$$

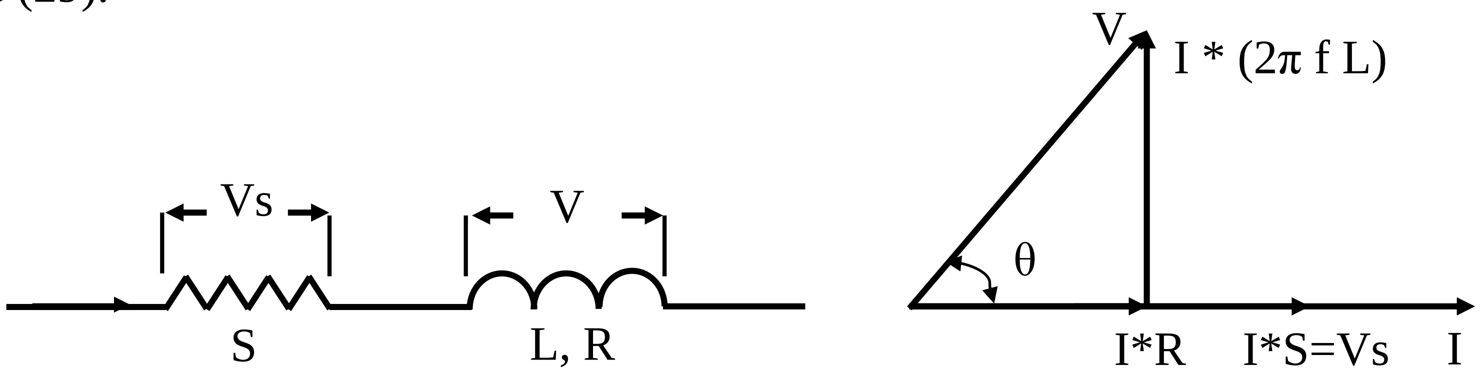


Figure (29) Ammeter-voltmeter method for inductance measurement.

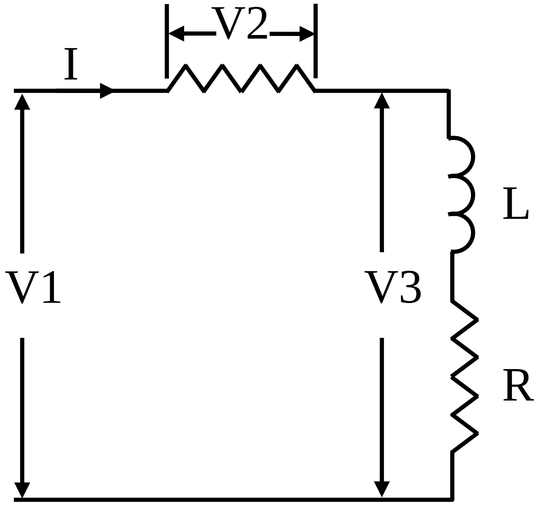
The phase angle θ can be known from the potentiometer readings. The other quantity to be measured is the supply frequency. For other quantity to be measured is the supply frequency. For the measurement of capacitance of a capacitor, an a.c. voltage of pure since wave is applied to the capacitor, and the voltage across it is measured by an electrostatic voltmeter. The current is measured by a low reading ammeter. Instead, a known non-inductive resistance may be connected in series with the capacitor and the voltage drop across it may be measured by a voltmeter, thus giving the current by the ratio of voltage drop to the series resistance.

If V and I be the voltage across and the current through the capacitor then the capacitance is given by

$$I = 2\pi f C V \qquad C = \frac{I}{2\pi f V}$$

2.2- The Three voltmeter method

The three voltmeter method for inductance measurement in shown in figure (30)



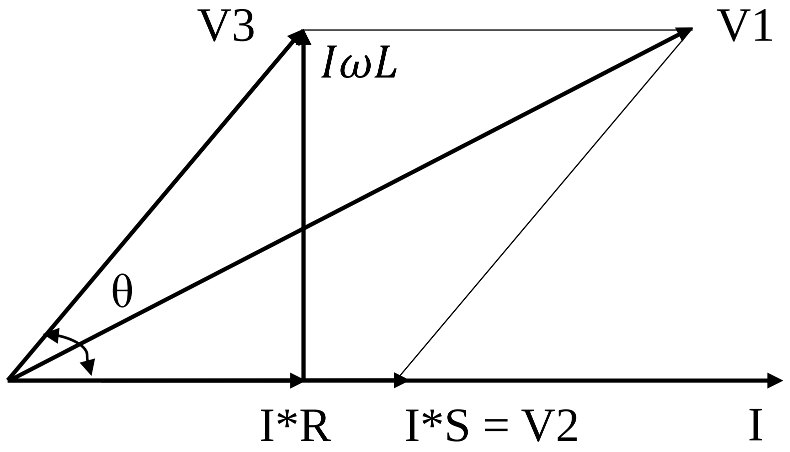
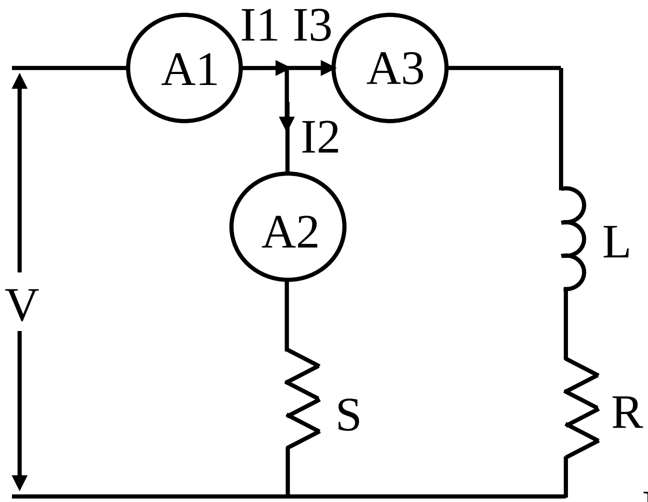


Figure (30)

2.3- The Three ammeter method.

The three ammeter method for inductance measurement in shown in figure (31)



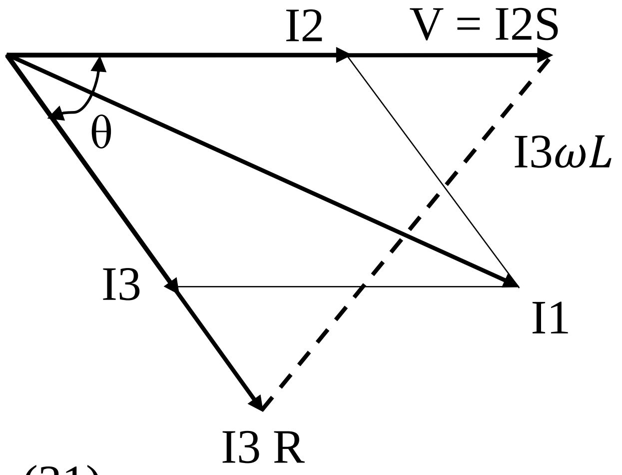


Figure (31)

2.4- AC bridge

Inductance and capacitance of AC circuit can also be quite accurately measured with bridge instruments.

2.4.1- General form of A.C. bridge

AC bridge are similar to D.C. bridge in topology (way of connecting). It consist of four arms AB,BC,CD and DA. Generally the impedance to be measured is connected between 'A' and 'B'. A detector is connected between 'B' and 'D'. The detector is used as null deflection instrument. Some of the arms are variable element. By varying these elements, the potential values at 'B' and 'D' can be made equal. This is called balancing of the bridge.

At the balance condition, the current through detector is zero.

A general form of AC bridge is shown in figure (32)

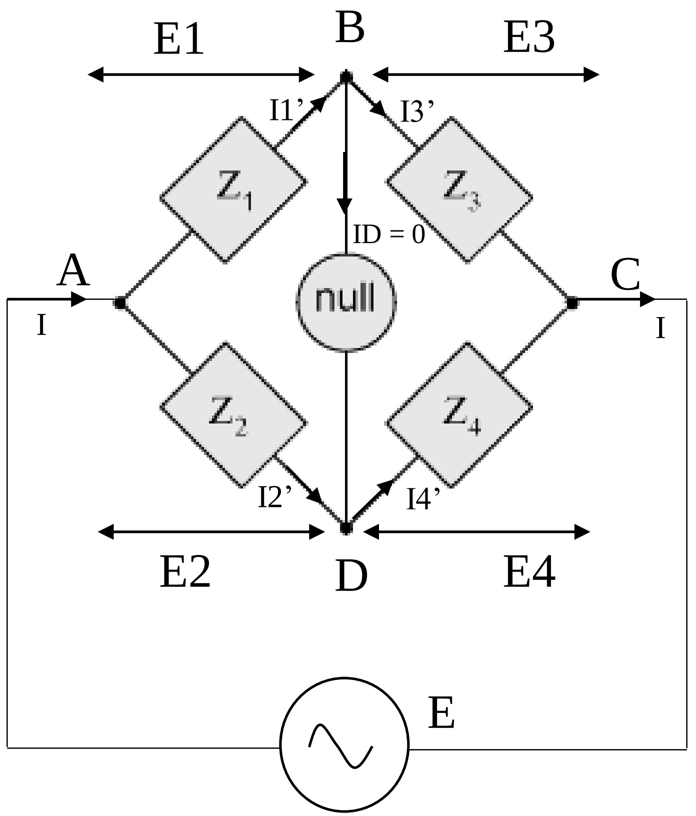


Figure (32) general form of AC bridge

$I1' = I3'$
$I2' = I4'$
$$\frac{I1'}{I2'} = \frac{I3'}{I4'}$$

At balance condition,
Voltage drop across ‘AB’=voltage drop across ‘AD’.
$E1' = E2' \quad and \quad E3' = E4'$

$Therefore \quad I1'Z1 = I2'Z2 \Rightarrow \frac{I1'}{I2'} = \frac{Z2'}{Z1'}$

$$I3'Z3 = I4'Z4 \Rightarrow \frac{I3'}{I4'} = \frac{Z4'}{Z3'}$$

$$\frac{Z2'}{Z1'} = \frac{Z4'}{Z3'} \Rightarrow Z2'\,Z3' = Z1'\,Z4'$$

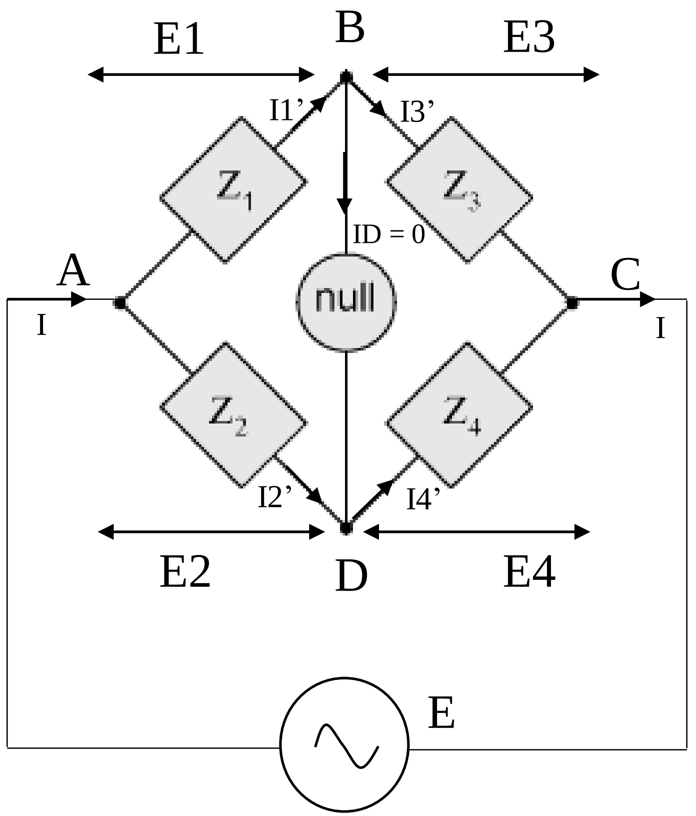


Figure (32) general form of AC bridge

$|Z2|\angle\theta 2\ |Z3|\angle\theta 3 = |Z1|\angle\theta 1 |Z4|\angle\theta 4$
$|Z2|\ |Z3| = |Z1|\ |Z4|$

$\theta 2 + \theta 3 = \theta 1 + \theta 4$

Example 4.3

- For balance condition, magnitude on either side must be equal.
- Angle on either side must be equal.

2.4.1.1- Types of detectors

The following types of instruments are used as detector in A.C. bridge.

A- Vibration galvanometer:

Between the point ‘B’ and ‘D’ a vibration galvanometer is connected to indicate the bridge balance condition. This A.C. galvanometer which works on the principle of resonance. The A.C. galvanometer shows a line “appears on the scale”, if the bridge is unbalanced.

B- Head phones

Two speakers are connected in parallel in this system. If the bridge is unbalanced, the speaker produced more sound energy. If the bridge is balanced, the speaker do not produced any sound energy.

C- Tuned amplifier

If the bridge is unbalanced the output of tuned amplifier is high. If the bridge is balanced, output of amplifier is zero.

2.5- Measurements of inductance
2.5.1- Maxwell's inductance bridge

The Maxwell bridge circuit is shown in figure (33).
The choke for which R1 and L1 have to measure connected between the points 'A' and 'B'. In this method the unknown inductance is measured by comparing it with the standard inductance.

L2 is adjusted, until the detector indicates zero current.
Let R1= unknown resistance
L1= unknown inductance of the choke.
L2= known standard inductance
R1,R2,R4= known resistances.

The quality factor (or Q) of an inductor is the ratio of its inductive reactance to its resistance at a given frequency, and is a measure of its efficiency

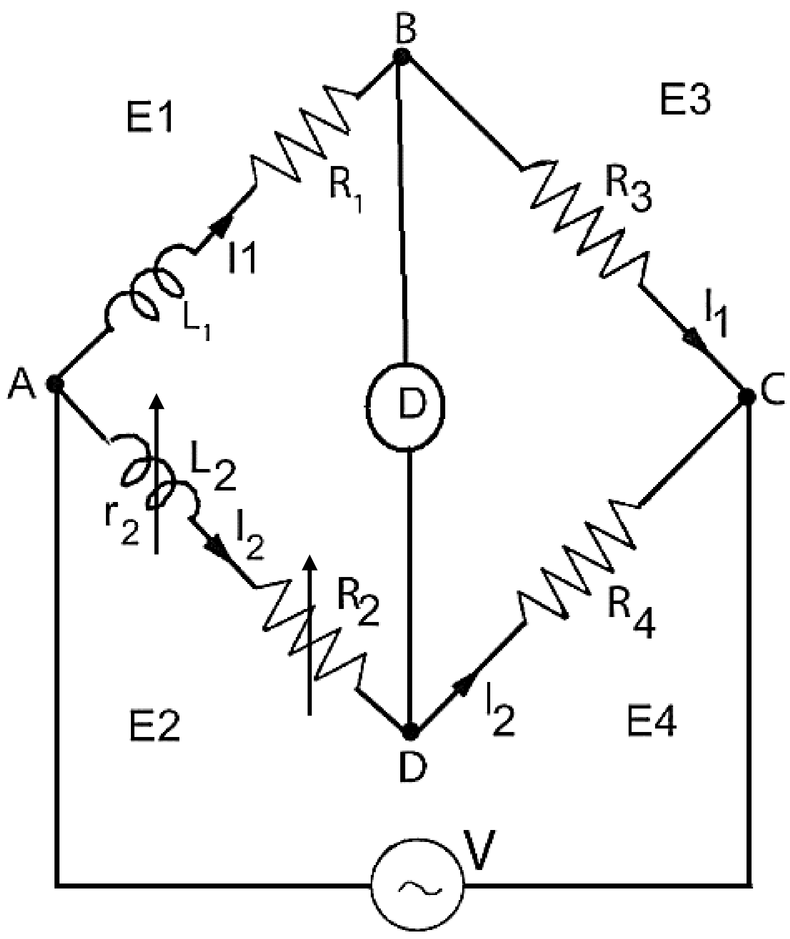


Figure (33) Maxwell's inductance bridge

At balance condition Z1' Z4' = Z2' Z3'

$$(R_1 + jXL_1)R_4 = (R_2 + jXL_2)R_3$$
$$(R_1 + j\omega L_1)R_4 = (R_2 + j\omega L_2)R_3$$
$$R_1R_4 + j\omega L_1R_4 = R_2R_3 + j\omega L_2R_3$$

Comparing real part,

$$R_1R_4 = R_2R_3 \Rightarrow R_1 = \frac{R_2R_3}{R_4}$$

Comparing the imaginary parts,

$$j\omega L_1R_4 = j\omega L_2R_3 \Rightarrow L_1 = \frac{L_2R_3}{R_4}$$

Q-factor of choke,

$$Q = \frac{\omega L_1}{R_1} = \frac{\omega L_2}{R_2}$$

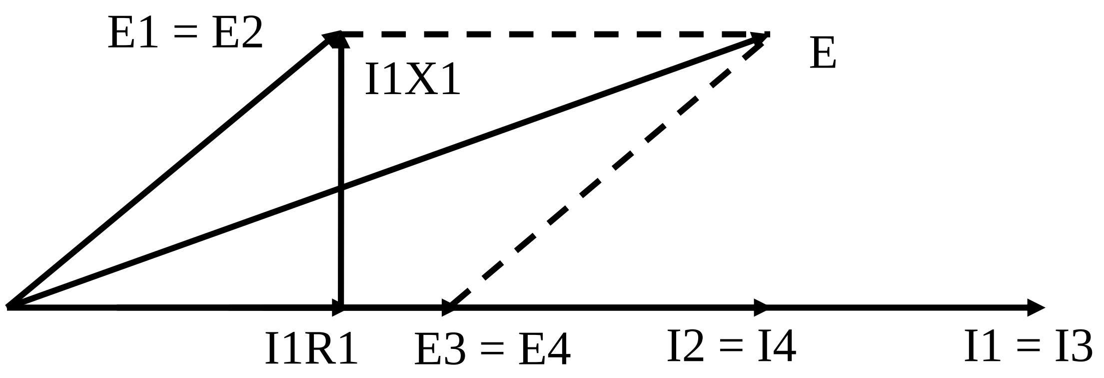


Figure (34) Phasor diagram of Maxwell's inductance bridge

Advantages

Expression for R1 and L1 are simple.
Equations area simple
They do not depend on the frequency (as w is cancelled)
R1 and L1 are independent of each other.

Disadvantages

Variable inductor is costly.
Variable inductor is bulky.

2.5.2- Maxwell's inductance capacitance bridge

Unknown inductance is measured by comparing it with standard capacitance. In this bridge, balance condition is achieved by varying 'C4'. Maxwell's inductance capacitance bridge circuit is shown in figure (35).

At balance condition, $Z_1Z_4 = Z_3Z_2$

$$Z_4 = R_4 \parallel \frac{1}{j\omega C_4} = \frac{R_4 * \frac{1}{j\omega C_4}}{R_4 + \frac{1}{j\omega C_4}} = \frac{R_4}{1 + j\omega C_4 R_4}$$

$$Z_1Z_4 = Z_3Z_2$$

$$(R_1 + j\omega L_1) * \frac{R_4}{1 + j\omega C_4 R_4} = R_3R_2$$

$$(R_1 + j\omega L_1)R_4 = R_3R_2(1 + j\omega C_4 R_4)$$

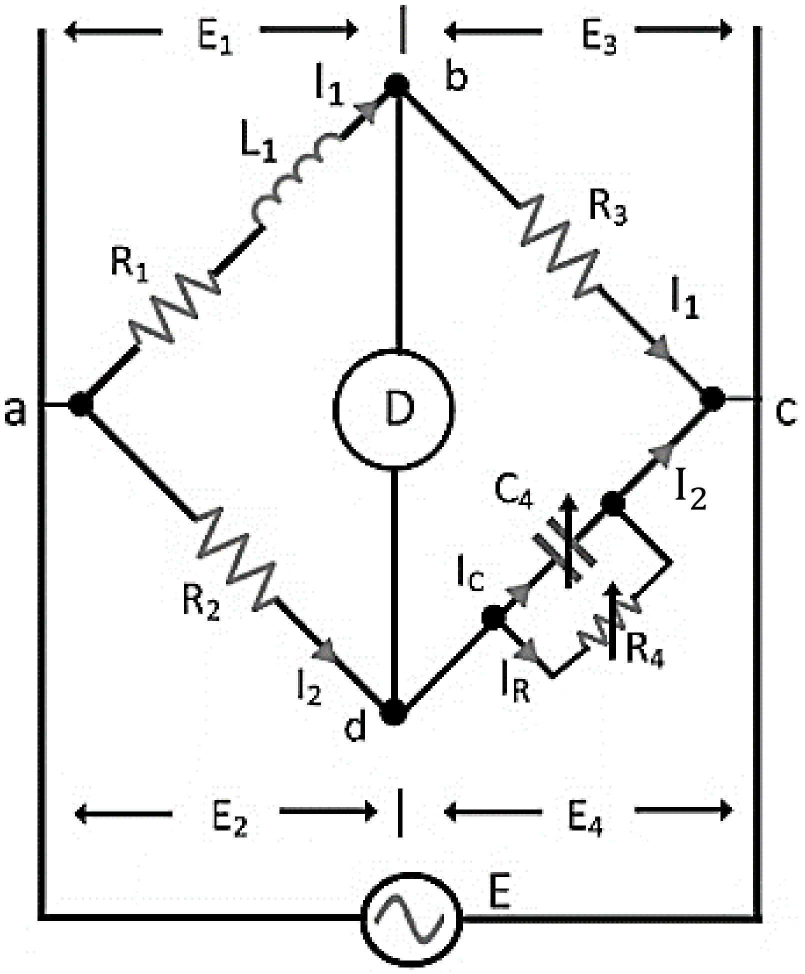


Figure (35) Maxwell's inductance capacitance bridge

$$(R_1 + j\omega L_1)R_4 = R_3R_2(1 + j\omega C_4R_4)$$
$$R_1R_4 + j\omega L_1R_4 = R_2R_3 + j\omega C_4R_4R_2R_3$$

Comparing real parts,

$$R_1R_4 = R_2R_3 \Longrightarrow R_1 = \frac{R_2R_3}{R_4}$$

Comparing imaginary part,

$$j\omega L_1R_4 = j\omega C_4R_4R_2R_3$$

$$L_1 = C_4R_2R_3$$

Q-factor of choke,

$$Q = \frac{\omega L_1}{R_1} = \omega C_4R_4$$

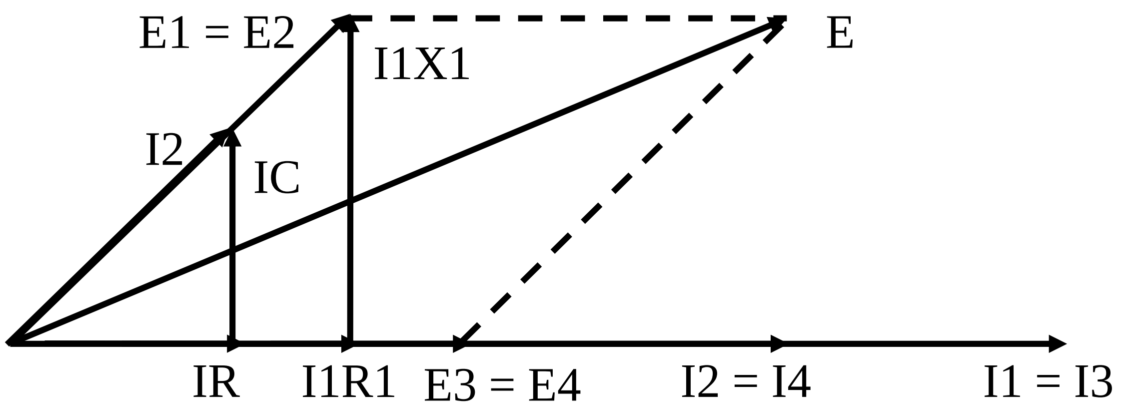


Figure (36) Phasor diagram of Maxwell's inductance capacitance bridge

Advantages

Equations of L1 and R1 are simple.
They are independent of frequency.
They are independent of each other.
Standard capacitor is much smaller in size than standard inductor.

Disadvantages

Standard variable capacitance is costly.
It can be used for measurements of Q-factor in the ranges of 1 to 10.
It cannot be used for measurements of choke with Q-factors more than 10.

We know that $Q = \omega C4R4$
For measuring chokes with higher value of Q-factor, the value of C4 and R4 should be higher. Higher values of standard resistance are very expensive. Therefore this bridge cannot be used for higher value of Q-factor measurements.

Example 4.4

2.5.3- Hay’s bridge
Hay’s bridge circuit and it’s phasor diagram are shown in figure (37).

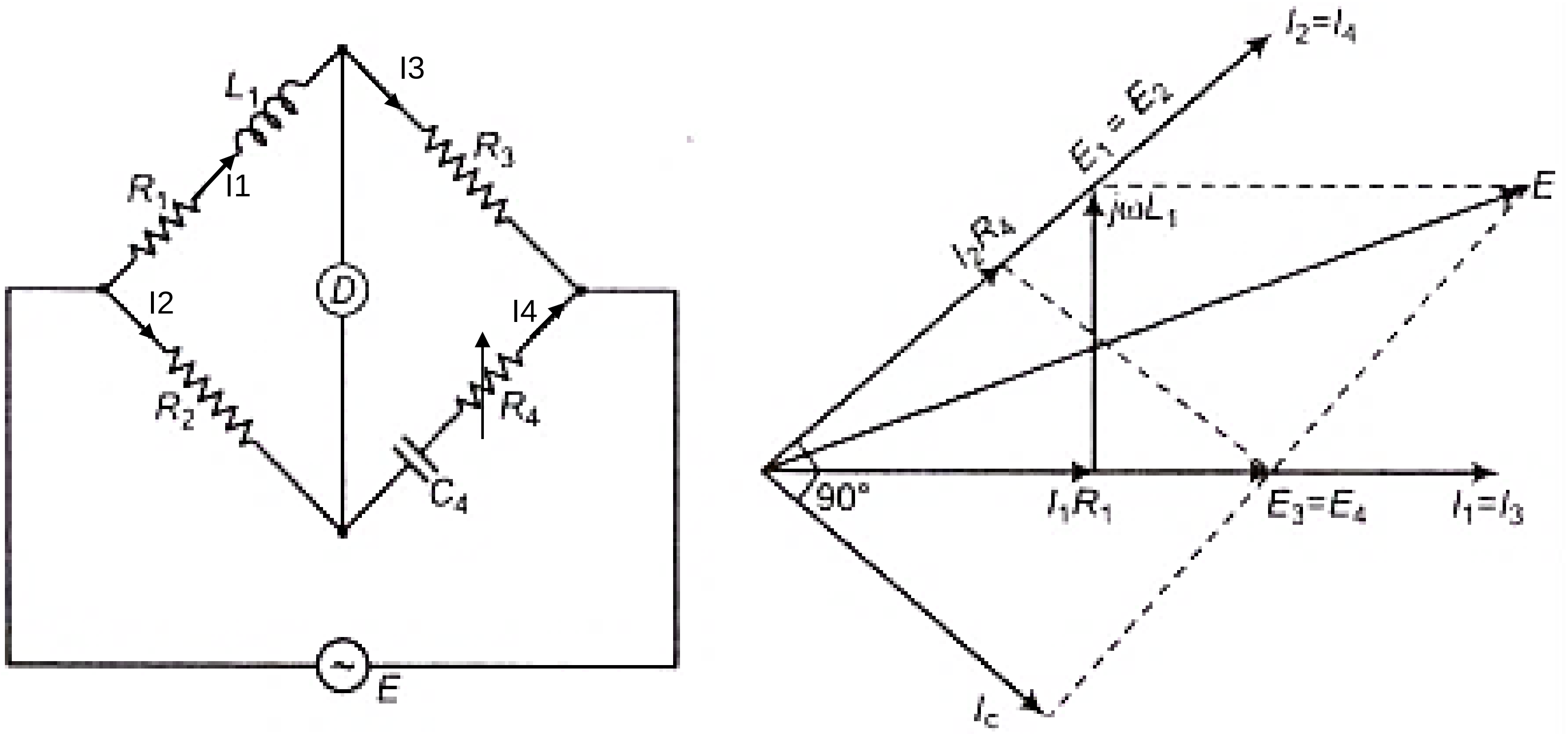


Figure (37) Hay’s bridge circuit and it’s phasor diagram

$E1' = I_1 R_1 + jI_1 X_1$
$E' = E1' + E3'$
$$E4' = I_4' R_4 + \frac{I_4}{j\omega C_4}$$
$E3' = I_3 R_3$
$$Z_4 = R_4 + \frac{1}{j\omega C_4} = \frac{1 + j\omega R_4 C_4}{j\omega C_4}$$

At balance condition, Z1 Z4=Z3 Z2

$$(R_1 + j\omega L_1)\frac{1 + j\omega R_4 C_4}{j\omega C_4} = R_2 R_3$$
$$(R_1 + j\omega L_1)(1 + j\omega R_4 C_4) = j\omega C_4 R_2 R_3$$
$$R_1 + j\omega L_1 + j\omega R_1 R_4 C_4 - \omega^2 L_1 C_4 R_4 = j\omega C_4 R_2 R_3$$
$$(R_1 - \omega^2 L_1 C_4 R_4) + j\omega(L_1 + R_1 R_4 C_4) = j\omega C_4 R_2 R_3$$

Comparing the real term,
$(R_1 - \omega^2 L_1 C_4 R_4) = 0$
$R_1 = \omega^2 L_1 C_4 R_4$

Comparing the imaginary terms,
$j\omega(L_1 + R_1 R_4 C_4) = j\omega C_4 R_2 R_3$
$L_1 = C_4 R_2 R_3 - R_1 R_4 C_4$

Homework 4.2 Drive R1 and L1

$$L_1 = \frac{C_4 R_2 R_3}{1 + \omega^2 {C_4}^2 {R_4}^2}$$

$$R_1 = \frac{\omega^2 {C_4}^2 R_2 R_3 R_4}{1 + \omega^2 {C_4}^2 {R_4}^2}$$

$$Q = \frac{\omega L_1}{R_1} = \frac{1}{\omega C_4 R_4}$$

Advantages

Fixed capacitor is cheaper than variable capacitor.

This bridge is best suitable for measuring high value of Q-factor.

Disadvantages

Equations of L1 and R1 are complicated.

Measurements of R1 and L1 require the value of frequency.

This bridge cannot be used for measuring low Q- factor.

Example 4.5

2.5.4- Owen's bridge

Owen's bridge circuit and it's phasor diagram are shown in figure (38).

$E_1 = I_1R_1 + jI_1X_1$
I_4 $leads$ E_4 by $90°$

$E' = E1' + E3'$

$E2' = I_2R_2 + \frac{I_2}{j\omega C_2}$

Balance condition, $Z_1'Z_4' = Z_2'Z_3'$

$Z_2 = R_2 + \frac{1}{j\omega C_2} = \frac{j\omega C_2R_2 + 1}{j\omega C_2}$

$(R_1 + j\omega L_1)\left(\frac{1}{j\omega C_4}\right) = \frac{(1 + j\omega R_2C_2)R_3}{j\omega C_2}$

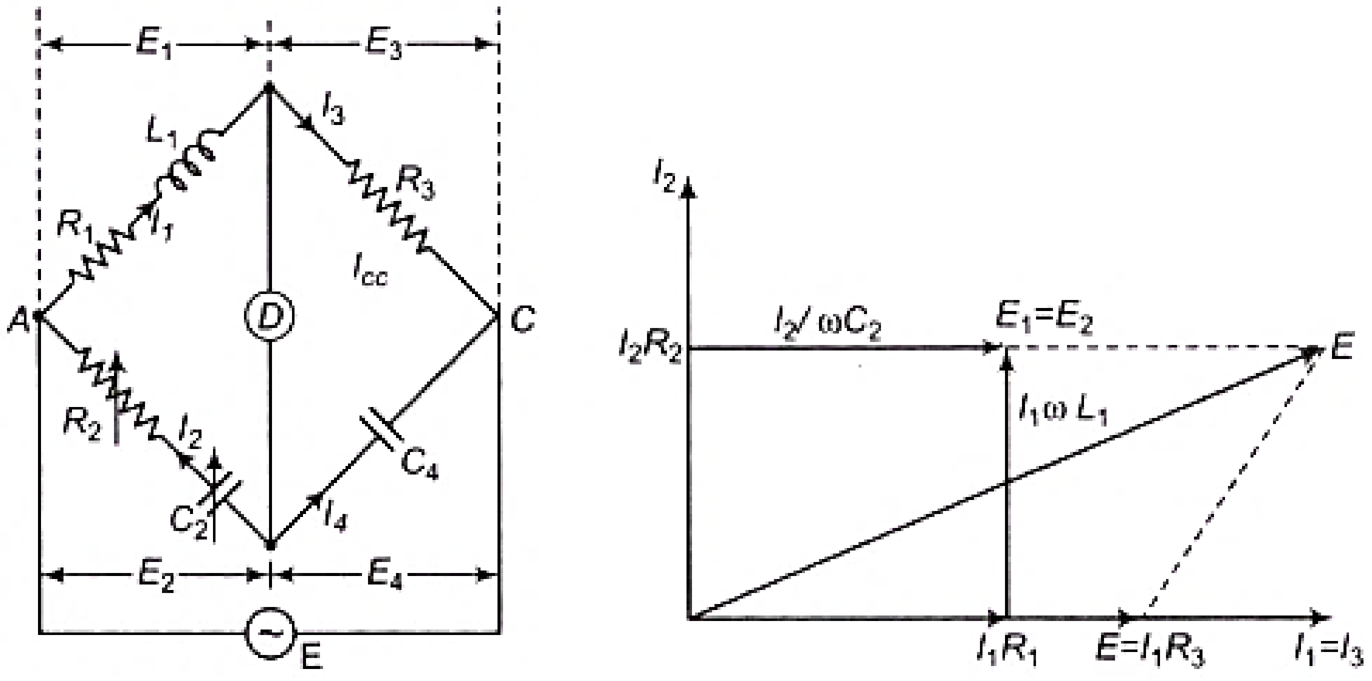


Figure (38) Owen's bridge circuit and phasor diagram

$$(R_1 + j\omega L_1)\left(\frac{1}{j\omega C_4}\right) = \frac{(1 + j\omega R_2 C_2)R_3}{j\omega C_2}$$
$$C_2(R_1 + j\omega L_1) = C_4(1 + j\omega R_2 C_2)R_3$$
$$C_2 R_1 + j\omega L_1 C_2 = C_4 R_3 + j\omega R_2 C_2 C_4 R_3$$

Comparing real terms,

$$C_2 R_1 = C_4 R_3 \Longrightarrow R_1 = \frac{C_4 R_3}{C_2}$$

Comparing imaginary terms,

$$j\omega L_1 C_2 = j\omega R_2 C_2 C_4 R_3$$
$$L_1 = C_4 R_2 R_3$$

$$Q - Factor = \frac{\omega L_1}{R_1} = \omega R_2 C_2$$

Advantages

Expression for R1 and L1 are simple.
R1 and L1 are independent of Frequency.

Disadvantages

The Circuits used two capacitors.
Variable capacitor is costly.
Q-factor range is restricted.

2.5.5- Anderson's bridge

Anderson's bridge circuit is shown in figure (39).

$$R_1 = \frac{R_2 R_3}{R_4} - r1$$

$$L_1 = R_3 C \left[\frac{R_2}{R_4}(r + R_4) + r \right]$$

Advantages

Variable capacitor is not required.
Inductance can be measured accurately.
R1 and L1 are independent of frequency.
Accuracy is better than other bridges.

Disadvantages

Expression for R1 and L1 are complicated.
This is not in the standard form A.C. bridge.

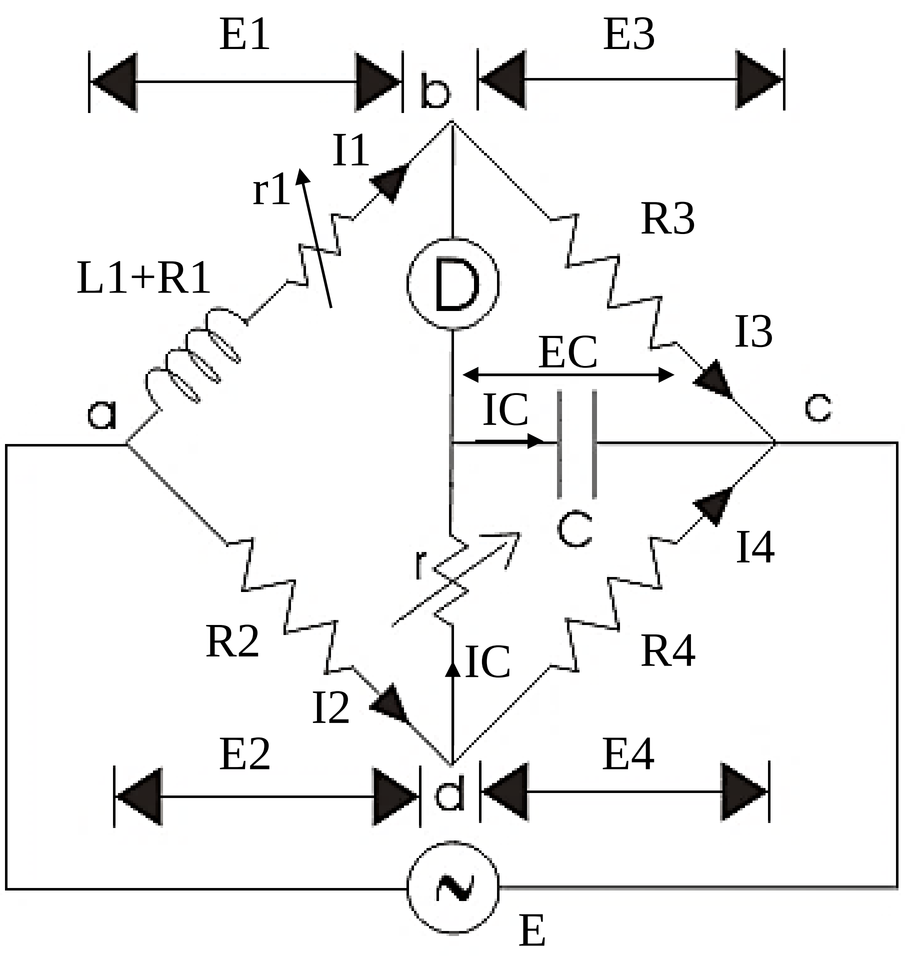


Figure (39) Anderson's bridge circuit

2.6- Measurements capacitance and loss angle. (Dissipation factor)

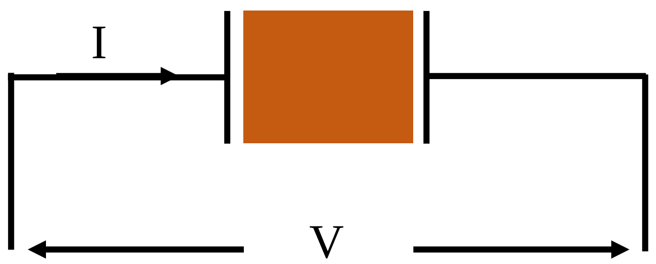


Figure (40) Condenser or capacitor

2.6.1-Dissipation factors (D)

A practical capacitor is represented as the series combination of small resistance and ideal capacitance. From the vector diagram, it can be seen that the angle between voltage and current is slightly less than 90°. The angle ‘δ’ is called loss angle.

A dissipation factor is defined as ‘tan δ’.

$$\tan\delta = \frac{IR}{IX_C} = \frac{R}{X_C} = \omega\, CR$$

$$D = \omega\, CR$$

$$D = \frac{1}{Q}$$

$$D = \tan\delta = \frac{\sin\delta}{\cos\delta} \quad \text{for small values of } \delta \text{ } \mathit{in\ radians}$$

$$D \cong \frac{\delta}{1} \cong \delta \cong \mathit{Loss\ Angle}\ (\delta\ \mathit{must\ be\ in\ radian})$$

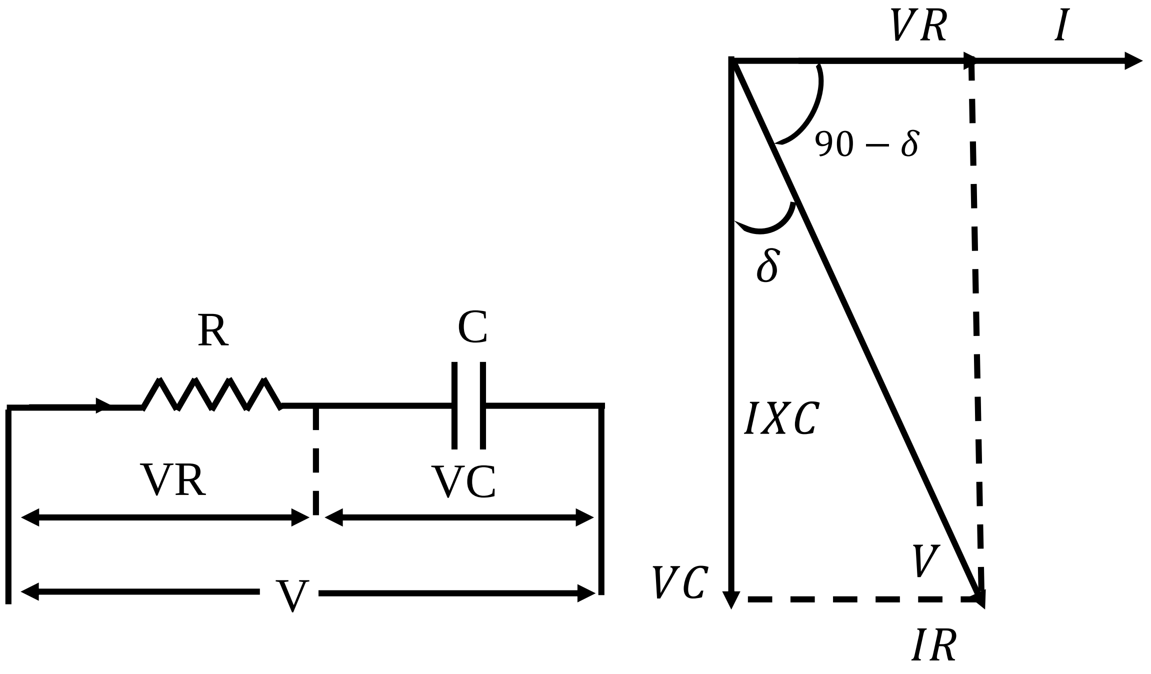


Figure (41) Representation of a practical capacitor and it’s Vector diagram

2.6.2- Desauty's Bridge

Desauty's bridge circuit is shown in figure (42)

$C1 = Unknown\ capacitance$

$At\ balance\ condition$

$$\frac{1}{j\omega C_1} * R_4 = \frac{1}{j\omega C_2} * R_3$$

$$\frac{R_4}{C_1} = \frac{R_3}{C_2} \Longrightarrow C_1 = \frac{R_4 C_2}{R_3}$$

Example 4.6

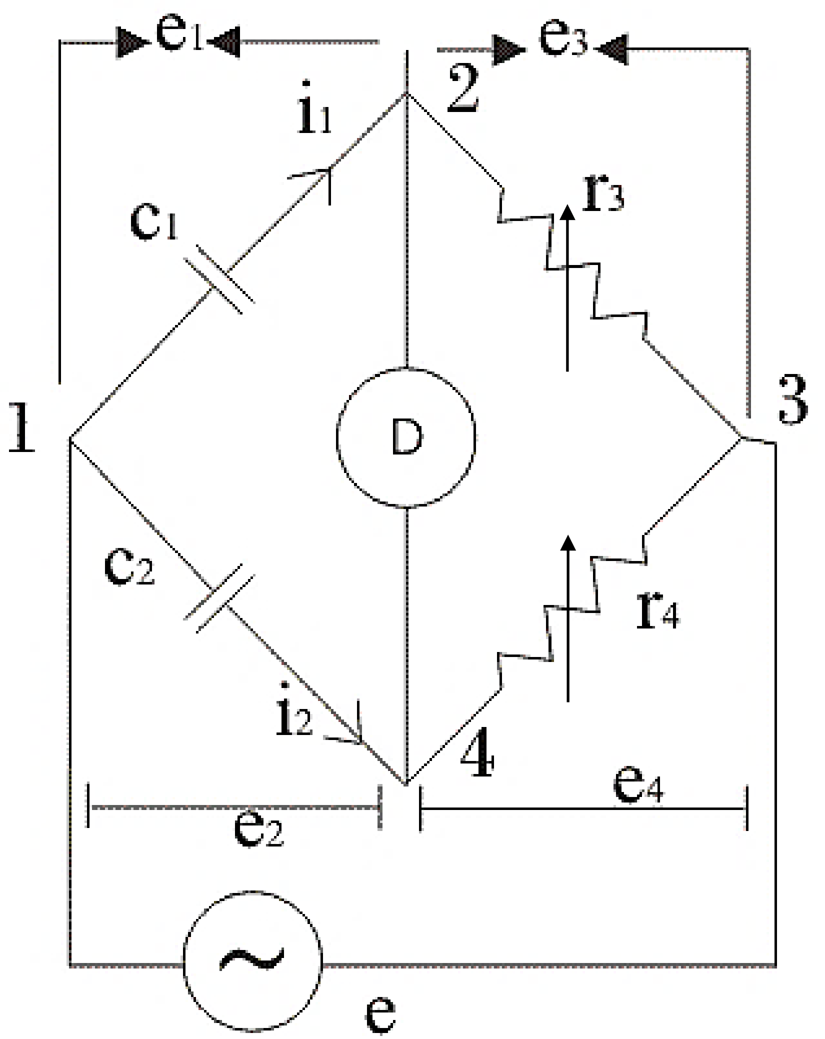


Figure (42) Desauty's Bridge

2.6.3- Schering bridge

Schering bridge circuit is shown in figure (43)

$E_1 = I_1 r_1 - jI_1 X_4$

$C_2 = C_4 = standard\ capacitor\ (internal\ resistacne = 0)$

$C_4 = Variable\ capacitance$

$C_1 = Unknown\ capacitance$

$r1 = Unknown\ series\ equivalent\ resistance\ of\ the\ capacitor$

$R_3 = R_4 = known\ resistor$

$$Z_1 = r_1 + \frac{1}{j\omega C_1} = \frac{j\omega C_1 r_1 + 1}{j\omega C_1}$$

$$Z_4 = \frac{R_4 * \frac{1}{j\omega C_4}}{R_4 + \frac{1}{j\omega C_4}} = \frac{R_4}{1 + j\omega C_4 R_4}$$

Homework 4.3

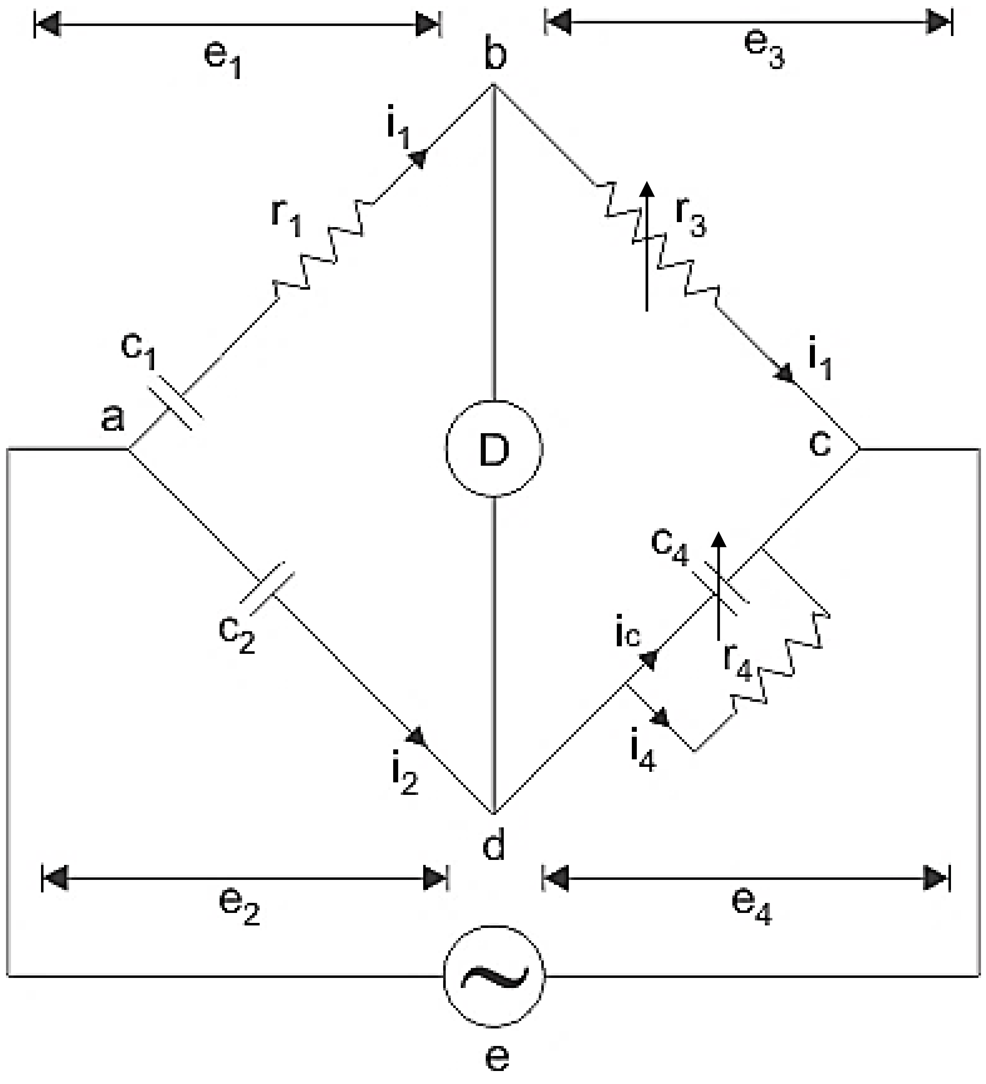


Figure (43) Schering bridge

At balance condition, Z1’ Z4’ = Z2’ Z3’

$$\frac{j\omega C_1 r_1 + 1}{j\omega C_1} * \frac{R_4}{1 + j\omega C_4 R_4} = \frac{R_3}{j\omega C_2}$$

$$(j\omega C_1 r_1 + 1) R_4 \, C_2 = R_3 \, C_1 (1 + j\omega C_4 R_4)$$

$$R_2 C_2 + j\omega C_1 r_1 R_4 C_2 = R_3 C_1 + j\omega C_4 R_4 R_3 C_1$$

Comparing the real part, $C_1 = \frac{R_4 C_2}{R_3}$

Comparing the imaginary part, $r_1 = \frac{C_4 R_3}{C_2}$

Dissipation factor of capacitor,

$$D = \omega C_1 r_1 = \omega * \frac{R_4 C_2}{R_3} * \frac{C_4 R_3}{C_2} \Longrightarrow D = \omega C_4 R_4$$

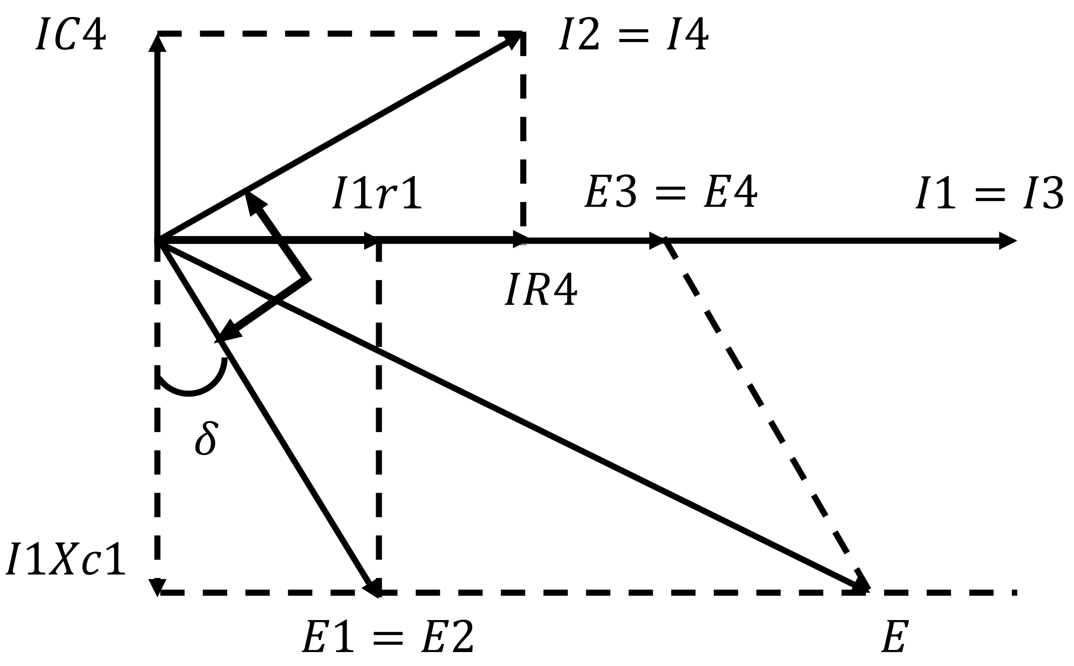


Figure (44) Phasor diagram of Schering bridge

Advantages

In this type of bridge, the value of capacitance can be measured accurately.

It can measure capacitance value over a wide range.

It can measure dissipation factor accurately.

Disadvantages

It requires two capacitors.

Variable standard capacitor is costly.

Chapter 5
Various measurements methods for determining frequency, phase angle, and power factor

1- Measurements of frequency

Frequency is the rate of occurrence of a repetitive event. If T is the period of a repetitive event, then the frequency $f = 1/T$. The international system of units (SI) states that the period should always be expressed in units of seconds (S), and the frequency should always be expressed in hertz (Hz).

1.1- Wein’s bridge
Wein’s bridge is popularly used for measurements of frequency. In this bridge, the value of all parameters are known. The source whose frequency has to be measured is connected as shown in figure (45).

$$Z_1 = r_1 + \frac{1}{j\omega C_1} = \frac{j\omega C_1 r_1 + 1}{j\omega C_1} \qquad Z_2 = \frac{R_2}{1 + j\omega C_2 R_2}$$

At balance condition, Z1’ Z4’ = Z2’ Z3’

$$\frac{j\omega C_1 r_1 + 1}{j\omega C_1} * R_4 = \frac{R_2}{1 + j\omega C_2 R_2} * R_3$$

$$(j\omega C_1 r_1 + 1)(1 + j\omega C_2 R_2) R_4 = j\omega C_1 R_2 R_3$$

$$[1 + j\omega C_2 R_2 + j\omega C_1 r_1 - \omega^2 C_1 C_2 r_1 R_2] = j\omega C_1 \frac{R_2 R_3}{R_4}$$

Comparing the real parts

$$1 - \omega^2 C_1 C_2 r_1 R_2 = 0$$

$$\omega^2 C_1 C_2 r_1 R_2 = 1$$

$$\omega^2 = \frac{1}{C_1 C_2 r_1 R_2}$$

$$\omega = \frac{1}{\sqrt{C_1 C_2 r_1 R_2}} \qquad f = \frac{1}{2\pi\sqrt{C_1 C_2 r_1 R_2}}$$

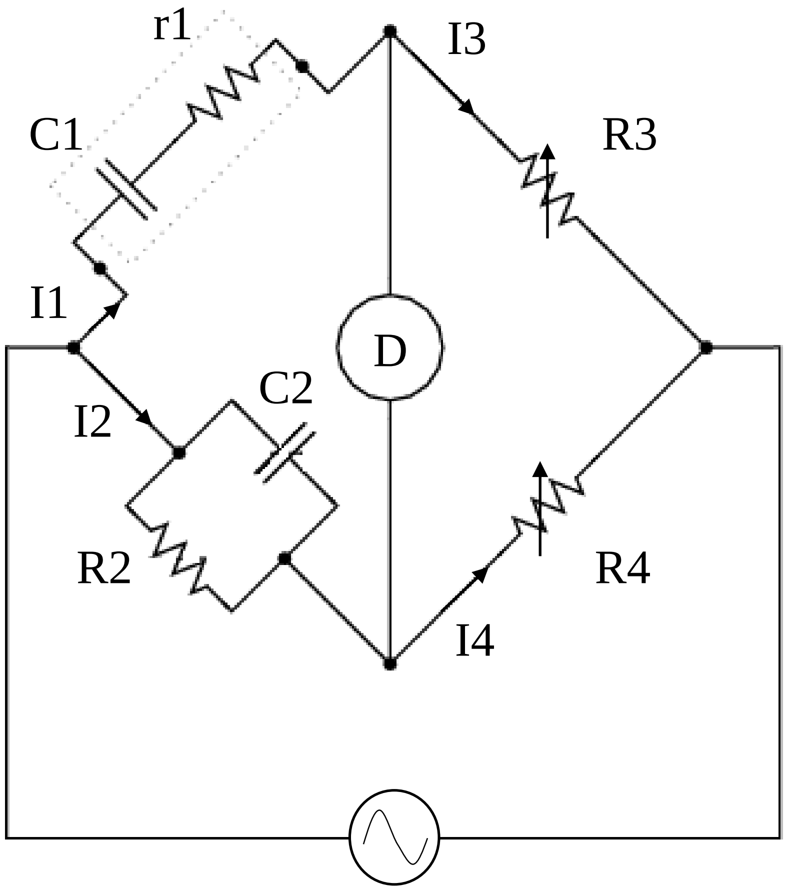


Figure (45) Wein’s bridge

2- Phase angle measurement:

The measurement of phase is important in almost all applications where sinusoidal proliferate. Many means have therefore been devised for this measurement. One of the most obvious measurement techniques is to directly measure the fractional part of the period that has been completed on a cathode-ray oscilloscope (CRO).

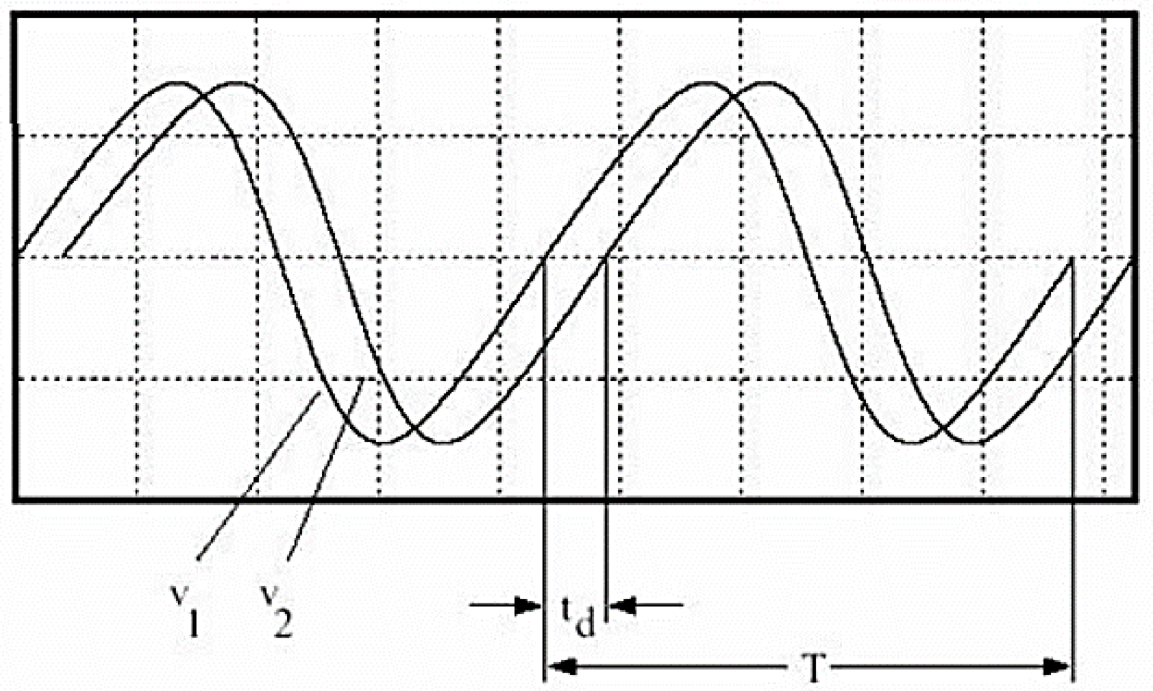


Figure (46) Dual-channel display. With either method, the sign of θ is determined by which channel is leading (to the left of) the other. In the figure, v1 leads v2.

2.1- Time difference method

Requirements:

- Two Oscilloscope channels

Method:

- Display both channels as a function of time.
- Scale each voltage channel so that each waveform fits in the display.
- Ground or zero each channel separately and adjust the line to the centre axis of the display.

- Return to ac coupling..
- Pick a feature (e.g., peak or zero crossing for sinusoids, rising or falling transition for square waves) to base your time measurements on. The peak of a sinusoid is not affected by dc offsets, but is harder to pinpoint than the zero crossing.

Then follow one of the methods below:	
Method 1	**Method 2** (requires continuous time base scaling)
• **Measure the period** T between repeats. Digital scopes often measure $f = 1 / T$ automatically. • **Measure** t_d, the smallest time difference between occurences of the feature on the two waveforms. • The phase difference is then $\lvert\theta_2 - \theta_1\rvert = 360° \frac{t_d}{T}$	• **Fit** one period of your waveform to 4, 6, or 9 divisions. • **Scale** the time base by a factor of ten (expand the plot horizontally), so that each division will be 9, 6, or 4°, respectively. • **Count** the number of divisions between similar points on the two waveforms.

3- Power factor measurement

The product of rms voltage and rms current does, however, define a quantity termed apparent power U.

$U = Erms\ Irms$ expressed in $volt - ampere$

If a delivered power, or real power, or active power is the power that does work, P

$P = Erms\ Irms\ Cos\ \varphi$ expressed in watts

Power factor is a single number that relates the active power, P, to the apparent power U.

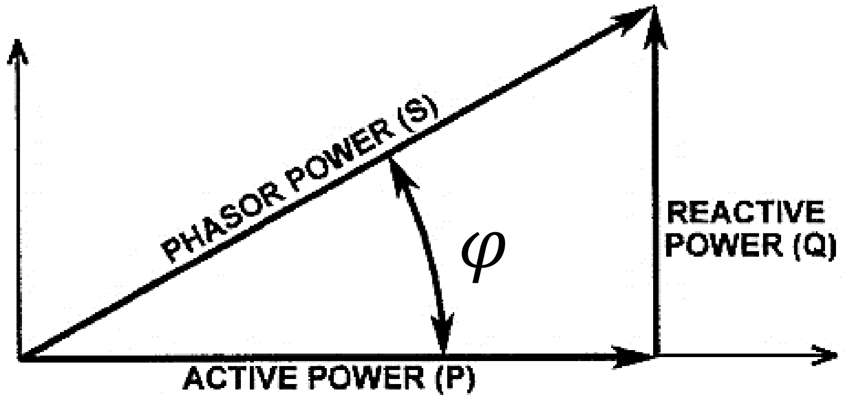


Figure (49) Power triangle.

$$PF = \frac{P}{U}$$

$$PF = Cos\ \varphi$$

There is something called a reactive power, or imaginary power, or wattles power, or magnetizing power, Q

$Q = Erms\ Irms\ sin\ \varphi$ expressed in voltamperes reactive, or VARS

If the load is predominantly inductive Q is positive
If the load is predominantly capacitive Q is negative

Phasor power $S = \sqrt{P^2 + Q^2}$

The power factor of an AC installation describes the phase relationship of the voltage and current. If the load is reactive, then the current is out of phase with the voltage.

When the voltage and current are sine waves, then the power factor is related to their phase relationship. The phase relationship can be measured by various specialized instruments, or an oscilloscope. For some loads, the current waveform is a complex shape, and then an oscilloscope is essential for measuring the shape of the current waveform. If the oscilloscope has the capability of spectrum analysis, then one can measure and calculate the harmonic content of the current. The power factor is then related to the magnitude of the fundamental component and the harmonics

When the load is resistive, the AC voltage and current are in phase and the power delivered to the load is the product of the two, in watts. Figure (47) shows an example of the voltage and current phase relationship for a resistive load.

When the load has some reactive component, then the current and voltage are no longer in phase. For example, if the load is an electric motor, then the coils of wire in the motor cause the current to lag the voltage. Figure (48) shows an example.

Figure (47) 60 Watt Lamp Load. The red trace is proportional to the AC line voltage. The blue trace is proportional to the current waveform. They are in phase because the load is resistive.

$$PF = Cos\ \varphi$$

$$\varphi = 360 * \frac{td}{T}$$

$$\varphi = 360 * \frac{0}{T} = 0$$

$$PF = 1$$

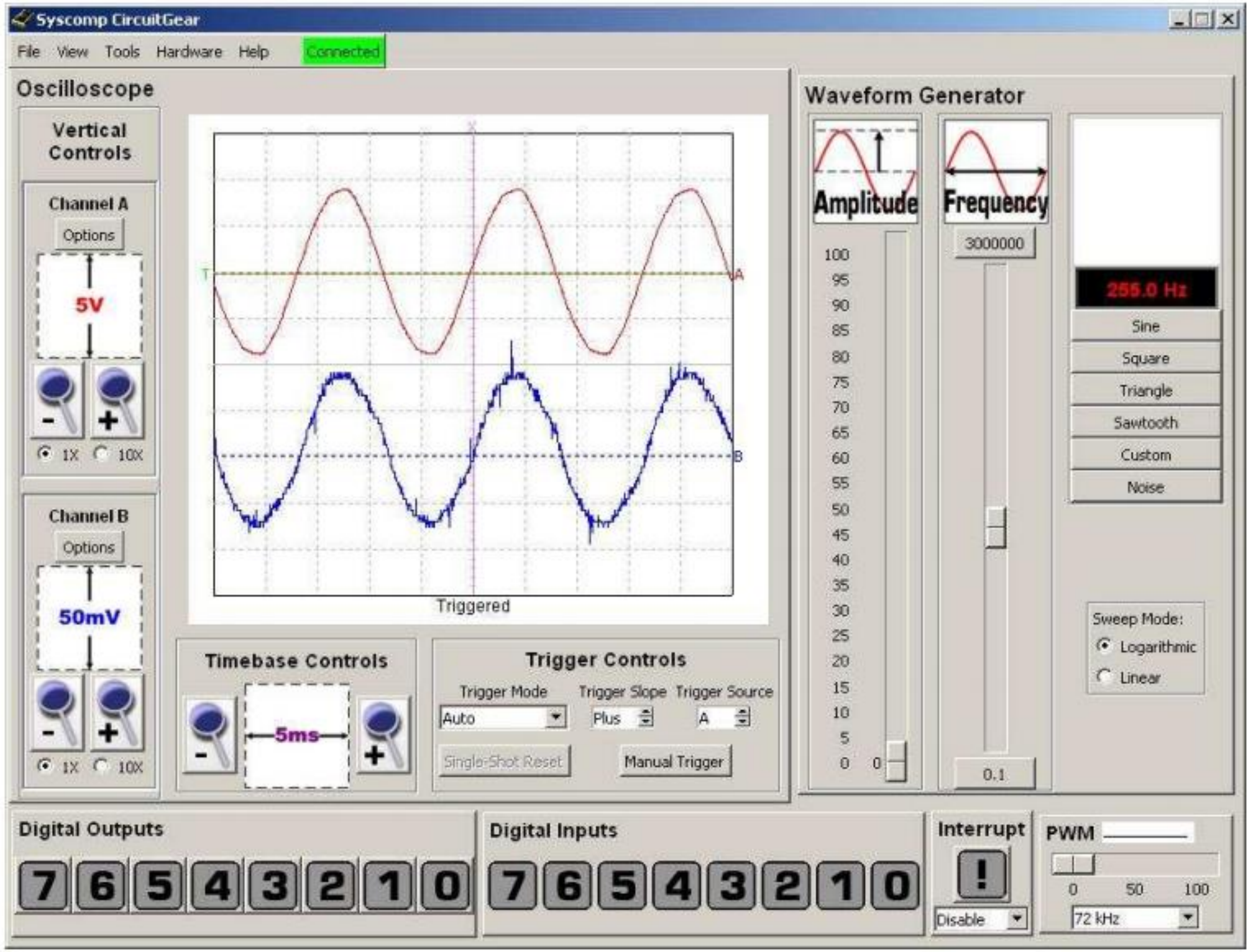
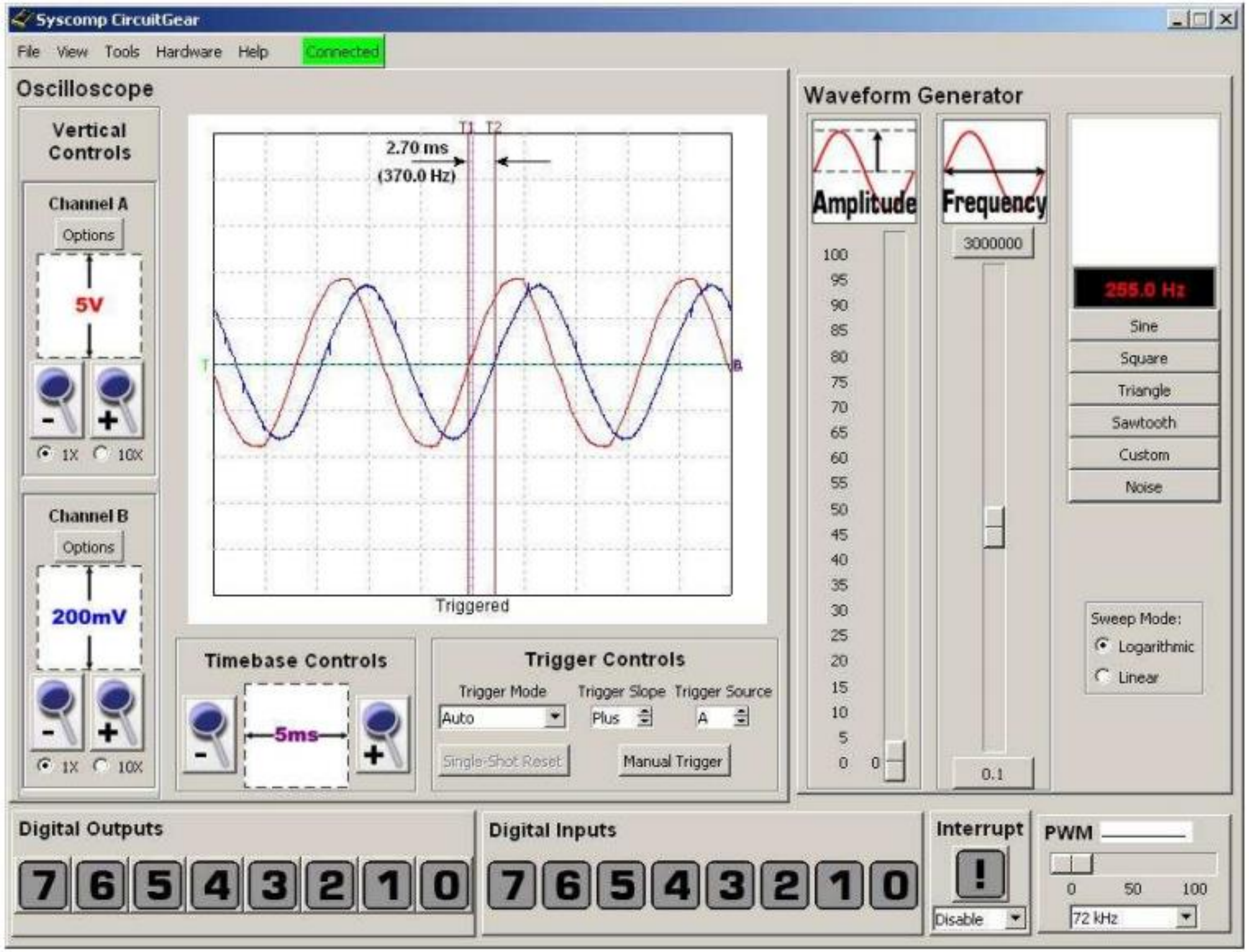


Figure (48) Electric Fan Load. The red trace is proportional to the AC line voltage. The blue trace is proportional to the current waveform. The current lags the voltage because the load is somewhat inductive. The time measurement cursors help determine the magnitude of the lagging phase angle.

For minimum reactive power, and to minimize the cost of electricity, the power factor should be close to unity, that is, the voltage and current should be in phase. For example, if the load is inductive, the line can be shunted by capacitors until the power factor is close to unity.



3.1- The three voltmeters method

Power factor meters exist, but are difficult to find and are hardly ever available on the home-brewer workbench. Even if you have an oscilloscope, it's still a tricky measurement to do: oscilloscopes are internally grounded and cannot be directly connected to the AC mains; floating the oscilloscope with an isolation transformer is a dangerous operation, since the scope chassis will be at mains potential. Then, the majority of the oscilloscopes do not withstand the direct mains voltage on their inputs and special high-voltage probes are required. On the other hand, if all these problems can be solved, measuring the angle φ with an oscilloscope is very accurate.

Fortunately, there is a very simple trick to measure *cos(φ)* called *the three voltmeters method*: you just need three AC voltmeters and a resistor. But in practice, you don't really need three voltmeters: you can get by with just one, and very often it's better to only use one.
The downside is that it only works fine for linear loads such as motors or transformers; it also works quite well with some slightly nonlinear loads like inductive fluorescent tube ballasts or transformer based arc welders, but doesn't work with strongly nonlinear loads such as rectifiers (basically any electronic ballast, switching power supply, motor driven by frequency converters,...). The idea is simple: just connect a resistor R in series with the load and measure the three voltages U_1, U_2 and U_3 as shown in this figure (49)

Once you have measured the three voltages U_1, U_2 and U_3, simply use the following equation to directly calculate the power factor:

$$PF = Cos\,\varphi = \frac{{U_1}^2 - {U_2}^2 - {U_3}^2}{2U_2U_3}$$

The actual value of R is not required for calculating the power factor, the voltage drop U2 across it is all you need

Example 5.1
Example 5.2

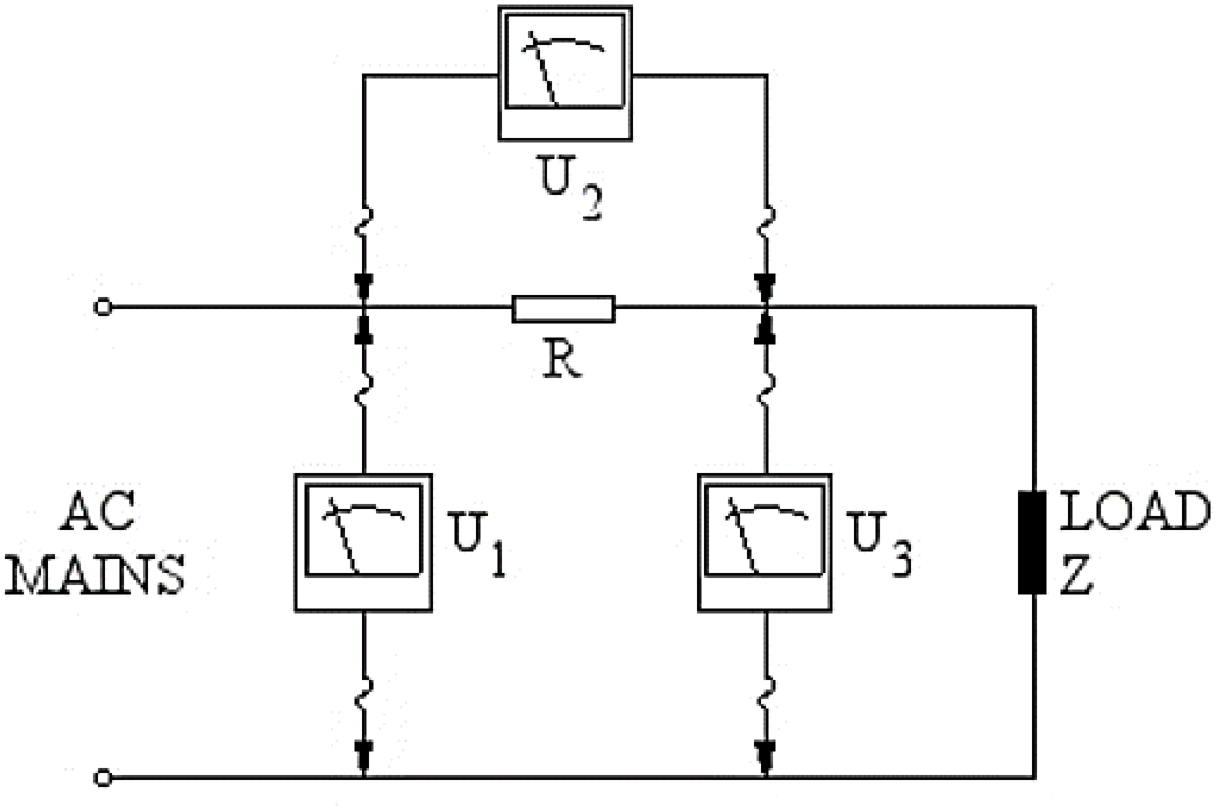


Figure (49) Connections of the three voltmeters and the additional resistor.

Chapter 6
Various measurements methods for determining hysteresis loop

6.1- Introduction:

The magnetic hysteresis loop reveals a lot of information about the properties of core materials used in ac machines and transformers. They indicate how high the core power losses are, what the maximum modulation may be and gives information about the drive dependent permeability.

Hysteresis loop is also called the B-H curve, “Flux density B against the magnetic flux intensity H”

The diagram of hysteresis loop is shown in figure (50)

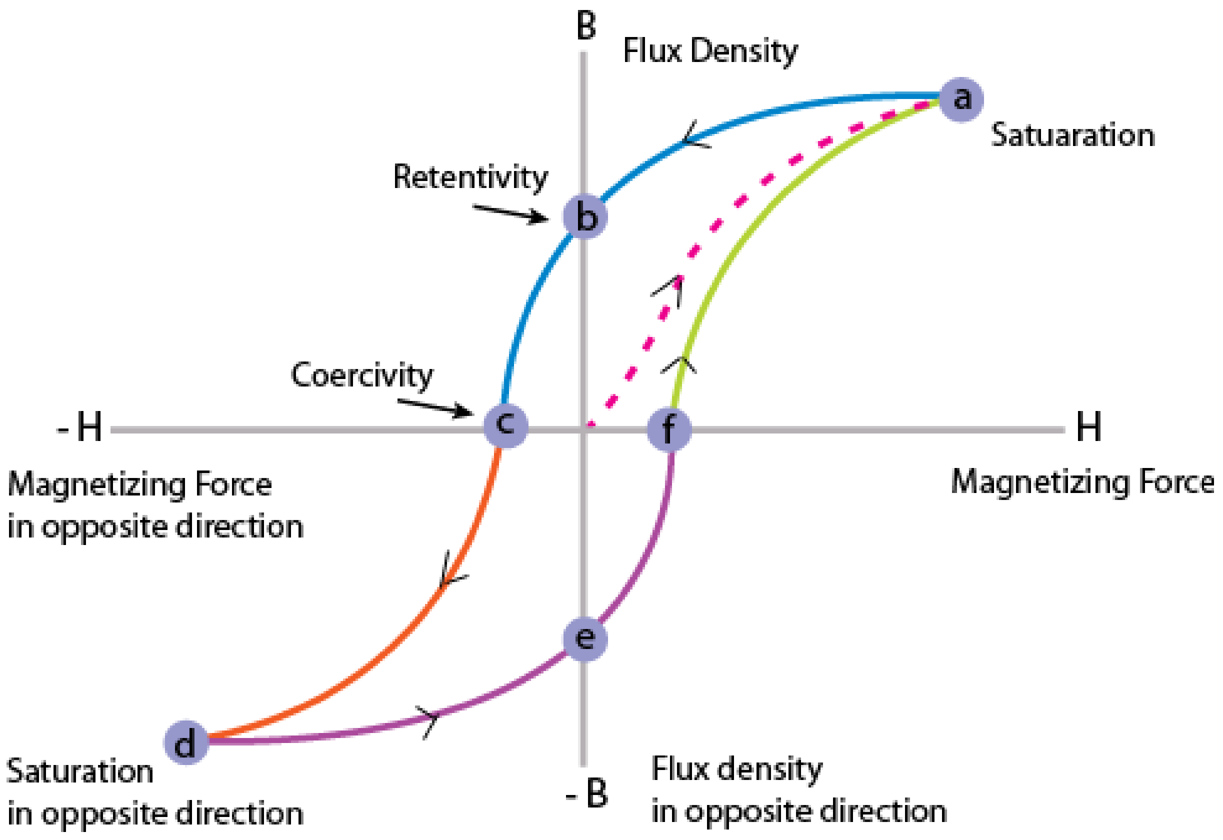


Figure (50) Hysteresis loop

- The magnetic material absorbs this energy each cycle and this energy is dissipated as heat.
- The amount of energy released per cycle is equal to the area of this hysteresis loop.
- To have a narrow "Thin" hysteresis loop we have to use a soft magnetic material. As shown in figure (51).
- To have a wide "Fat" hysteresis loop we have to use a hard magnetic material. As shown in figure (51).

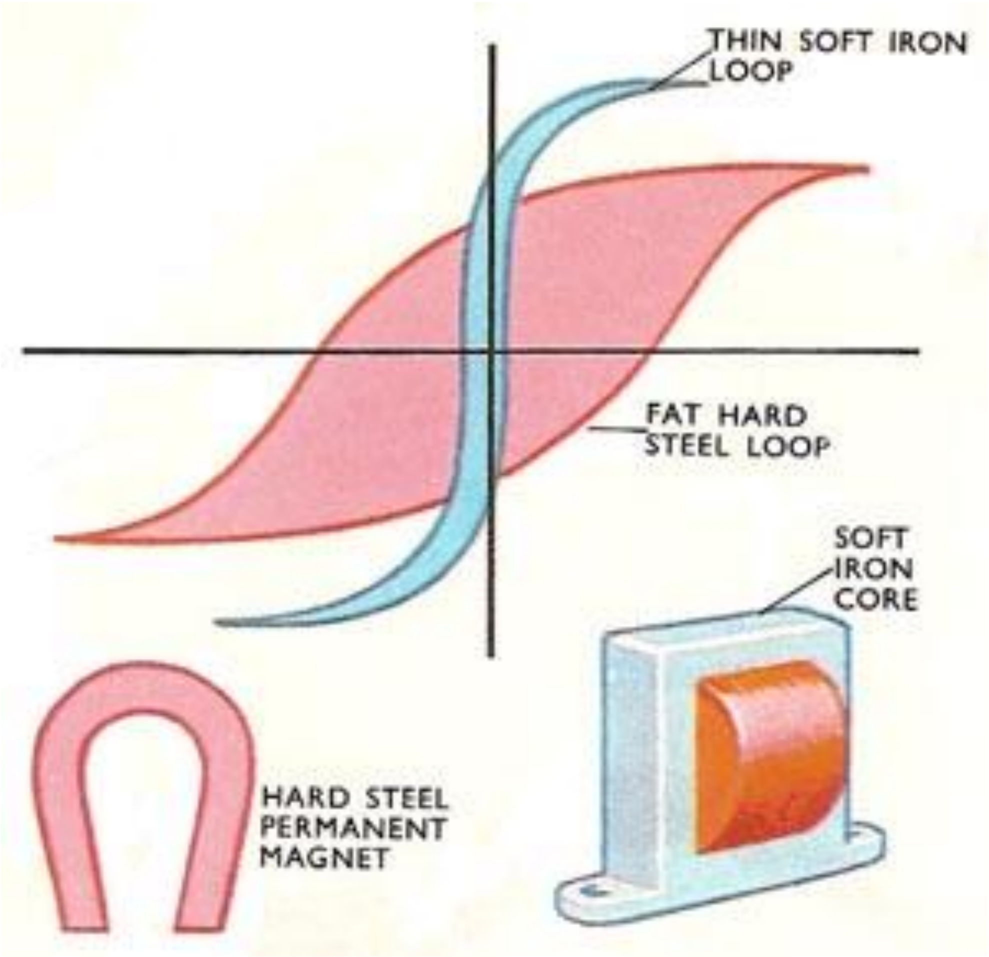


Figure (51) Hard and Soft "Fat and Narrow hysteresis loop"

Hysteresis loss can be calculated using an empirical formula:

$$P_h = K_h * V * f * B^n{}_{max}$$

Where:

P_h is the hysteresis loss in (watt)

K_h is the hysteresis constant depends on the magnetic material

V is the material volume

f is the supply frequency

B_{max} is the maximum flux density

n is the material constant ranging between (1.5 - 2.5)

6.2- Measuring the hysteresis loop "BH curve"

An easy way to measure the BH curve is by using CRO, cathode ray oscilloscope. Let us assume that we are trying to measure hysteresis losses of core material of a transformer. To measure the BH curve you have to use 2 channels. We cannot "measure" B and H directly. Further if we have transformer, we only have terminal measurements at our disposal. Hence, it is required to "process" the signals to get values of B and H. From Faraday's law,

$V = N\frac{d\phi}{dt}$ also from equation $\phi_m = B_m\, A_c$

B is directly proportional to flux ϕ. A signal proportional to B can be obtained by integrating the voltage signal. The voltage can be integrated approximately by using an RC circuit. Care should be taken in the choice of R and C values. The transfer function of the circuit is given by:

$$\frac{V_C(s)}{V_{in}(s)} = G(s) = \frac{1}{1 + s\tau}$$

The time constant $\tau = RC$ should be chosen such that, in frequency domain $(1 + j\omega\tau) \approx j\omega\tau$

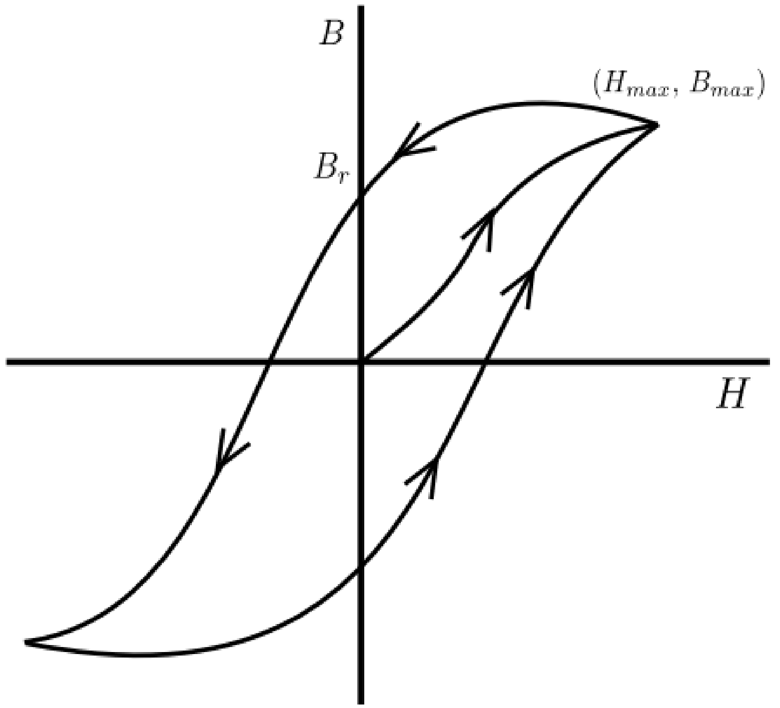


Figure (52) Hysteresis loop

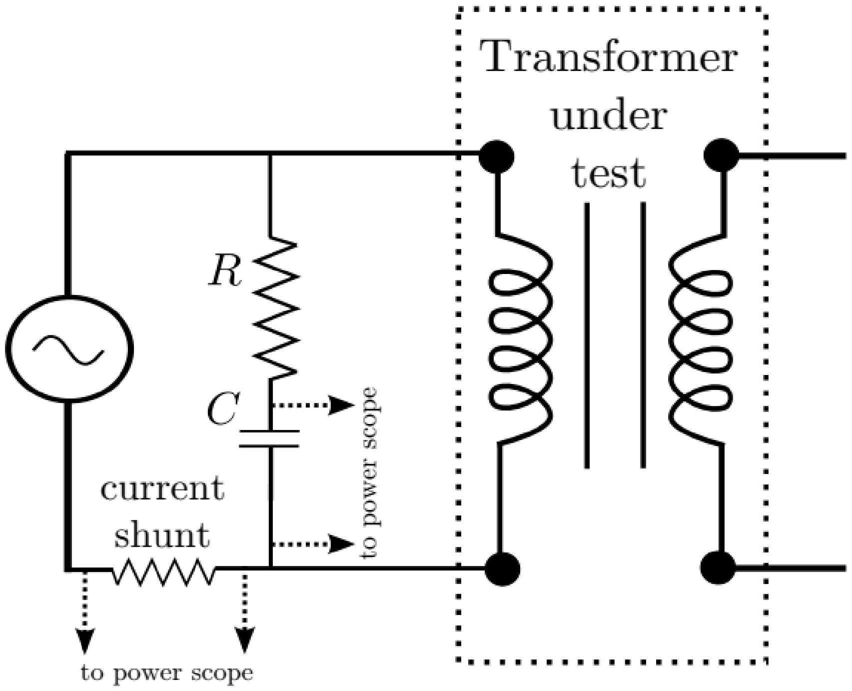


Figure (53) Connection diagram of circuit to trace B − H curve measurement

If a current carrying coil produces magnetic flux which traverses an average length of l in complete flux path, then: $Hl = NI$ tells that H is proportional to current. The two signals can be given to two channels of a digital storage oscilloscope. B − H curve of the material can be seen by plotting using Lissajous plot settings of the oscilloscope.

In Lissajous plot the signals from two channels are plotted against each other. Otherwise, each signal is plotted against time.

6.2- Epstein Meter

Another way to visualize B − H curve of the material is by using Epstein Meter. This meter is used to visualize the B − H curve of a material which is available in form laminations. Laminations of the available material can be inserted in the apparatus and the B−H curve can be seen on the oscilloscope. The strips are inserted in the meter. The instrument is constructed such that the inserted strips form core of a transformer. No load losses in the transformer are calculated to plot B-H curve of the material.

Figure (54) Digital Epstein tester VET-1608 Epstein Bridge VEB-25

Chapter 7
High voltage measurements and testing

7.1- Definition of High voltage

In electric power transmission engineering, high voltage is usually considered any voltage over approximately 35,000 volts. This is a classification based on the design of apparatus and insulation.

The International Electrotechnical Commission and its national counterparts (IET, IEEE, VDE, etc.) define high voltage as above 1000 V for alternating current, and at least 1500 V for direct current, and distinguish it from low voltage (50–1000 V AC or 120–1500 V DC) and extra-low voltage (<50 V AC or <120 V DC) circuits. This is in the context of building wiring and the safety of electrical apparatus. Voltages over approximately 50 volts can usually cause dangerous amounts of current to flow through a human being who touches two points of a circuit, so safety standards, in general, are more restrictive around such circuits.

Figure (55) High Voltage sign

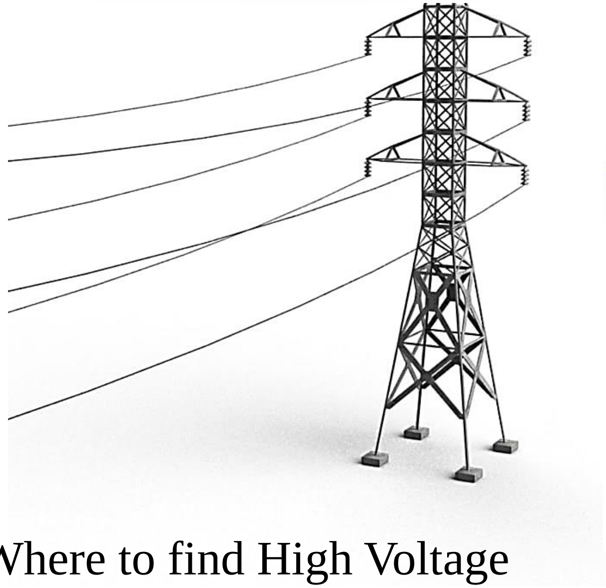
7.2- Where to find High Voltage

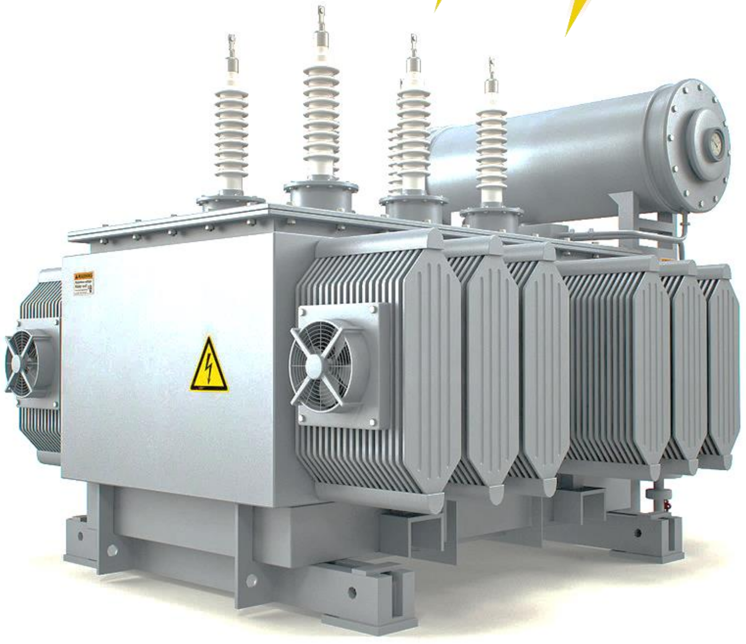
HV testing units

Power plants

Lightning

Transmission and distribution systems





HV transformers

Figure (56) Where to find High Voltage

Although the majority of the world's electric transmission is carried on a.c. systems, high-voltage direct current (HVDC) transmission by overhead lines, submarine cables, and back-to-back installations provides an attractive alternative for bulk power transfer. HVDC permits a higher power density on a given right-of-way as compared to a.c. transmission and thus helps the electric utilities in meeting the environmental requirements imposed on the transmission of electric power. HVDC also provides an attractive technical and economic solution for interconnecting asynchronous a.c. systems and for bulk power transfer requiring long cables.

7.3- High voltage testing

7.3.1- Testing with power frequency voltages

To assess the ability of the apparatus's insulation withstand under the system's power frequency voltage the apparatus is subjected to the 1-minute test under 50 Hz or 60 Hz depending upon the country. The test voltage is set at a level higher than the expected working voltage in order to be able to simulate the stresses likely to be encountered over the years of service. For indoor installations the equipment tests are carried out under dry conditions only. For outdoor equipment tests may be required under conditions of standard rain as prescribed in the appropriate standards.

Figure (57) Testing equipment for testing with power frequency voltages

7.3.2- Testing with lightning impulse voltages

Lightning strokes terminating on transmission lines will induce steep rising voltages in the line and set up travelling waves along the line and may damage the system's insulation. The magnitude of these over-voltages may reach several thousand kilovolts, depending upon the insulation. Exhaustive measurements and long experience have shown that lightning over-voltages are characterized by short front duration, ranging from a fraction of a microsecond to several tens of microseconds and then slowly decreasing to zero. The standard impulse voltage has been accepted as an aperiodic impulse that reaches its peak value in 1.2 μsec and then decreases slowly (in about 50 μsec) to half its peak value. Full details of the waveshape of the standard impulse voltage together with the permitted tolerances are shown in figure (58). In addition to testing equipment, impulse voltages are extensively used in research laboratories in the fundamental studies of electrical discharge mechanisms, notably when the time to breakdown is of interest.

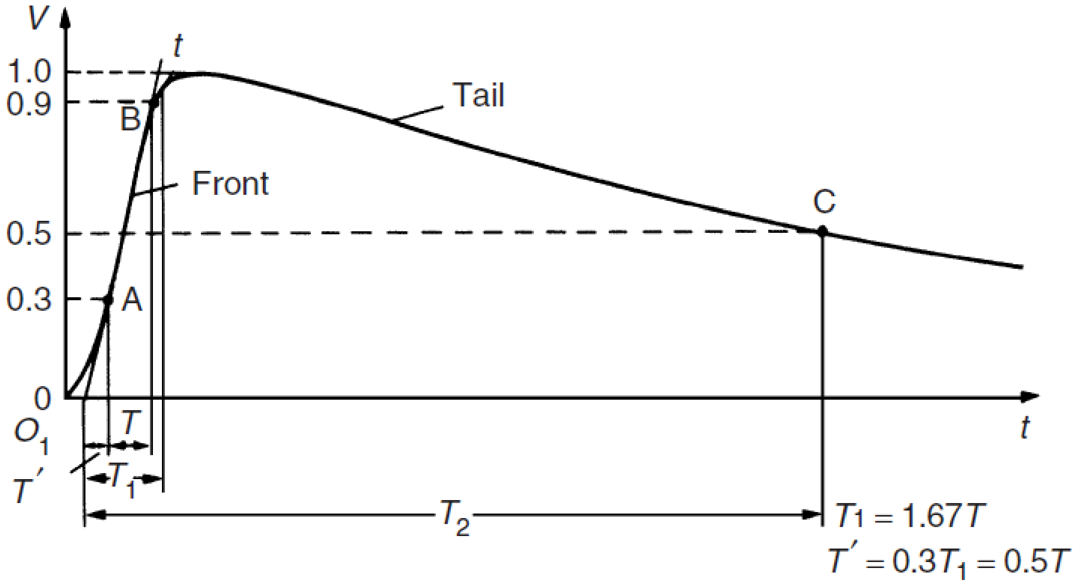


Figure (58) Standard impulse waveshape

7.3.3- Testing with switching impulses

Transient over-voltages accompanying sudden changes in the state of power systems, e.g. switching operations or faults, are known as switching impulse voltages. It has become generally recognized that switching impulse voltages are usually the dominant factor affecting the design of insulation in h.v. power systems for rated voltages of about 300 kV and above. Accordingly, the various international standards recommend that equipment designed for voltages above 300 kV be tested for switching impulses. Although the waveshape of switching over-voltages occurring in the system may vary widely, experience has shown that for flashover distances in atmospheric air of practical interest the lowest withstand values are obtained with surges with front times between 100 and 300 μsec. Hence, the recommended switching surge voltage has been designated to have a front time of about 250 μsec and half value time of 2500 μsec. For GIS (gas-insulated switchgear) on-site testing, oscillating switching impulse voltages are recommended for obtaining higher efficiency of the impulse voltage generator.

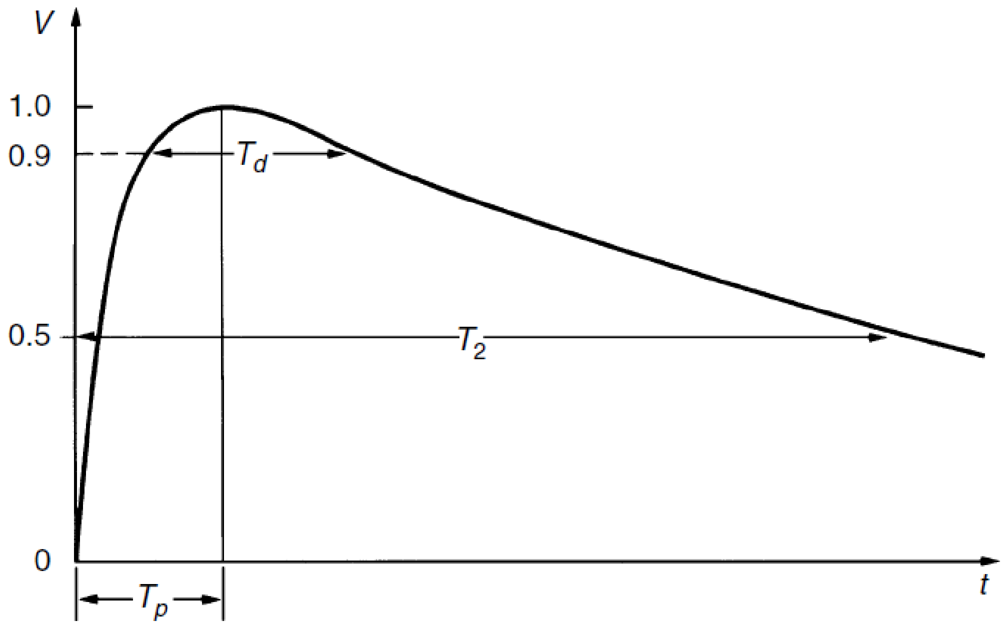


Figure (59) switching impulses

7.3.4- Testing with very low-frequency voltage

In the earlier years when electric power distribution systems used mainly paper-insulated lead covered cables (PILC) shown in figure (60) on-site testing specifications called for tests under d.c. voltages. Typically the tests were carried out at 4 – 4.5 V. The tests helped to isolate defective cables without further damaging good cable insulation. With the widespread use of extruded insulation cables of higher dielectric strength, the test voltage levels were increased to 5 – 8 V. In the 1970s premature failures of extruded dielectric cables factory tested under d.c. voltage at specified levels were noted1. Hence on-site testing of cables under very low frequency (VLF) of ~ 0.1Hz has been adopted.

Figure (60) paper-insulated lead covered cables (PILC)

7.4- Measurement of high voltages

Definitions of some terms:

The terms "uncertainty", "error" and "tolerance" are often mixed up. Therefore, the following clarification seems needed: The uncertainty is a parameter which is associated with the result of a measurement. It characterizes the dispersion of the results due to the characteristics of the measuring system. The error is the measured quantity minus a reference value for this quantity and the tolerance is the permitted difference between the measured and the specified value. Tolerances play a role for standard HV test procedures. Uncertainties are important for the decision, whether a measuring system is applicable or not for acceptance testing.

Precise measurement of high test voltages is considered to be a difficult task for many years.

A HV measuring system(MS) is a "complete set of devices suitable for performing a HV measurement". Software for the calculation of the result of the measurement is a part of the measuring system (IEC 60060-2:2010).

A HV measuring system shown in Figure (61) which should be connected directly to the test object consists usually of the following components

1- A converting device including its HV and earth connection to the test object which converts the quantity to be measured (measured: test voltage with its voltage and/or time parameters) into a quantity compatible with the measuring instrument (low-voltage or current signal). It is very often a voltage divider of a type depending on the voltage to be measured. For special application also a voltage transformer, a voltage converting impedance (carrying a measurable current) or an electric field probe (converting amplitude and time parameters of an electric field) may be used.

2- A transmission system which connects the output terminals of the converting device with the input terminals of the measuring instrument. It is very often a coaxial cable with its terminating impedance, but may also be an optical link which includes a transmitter, an optical cable and a receiver with an amplifier.

3- A measuring instrument suitable to measure the required test voltage parameters from the output signal of the transmission system. Measuring instruments for HV application are usually special devices which fulfil the requirements of the IEC Standards.

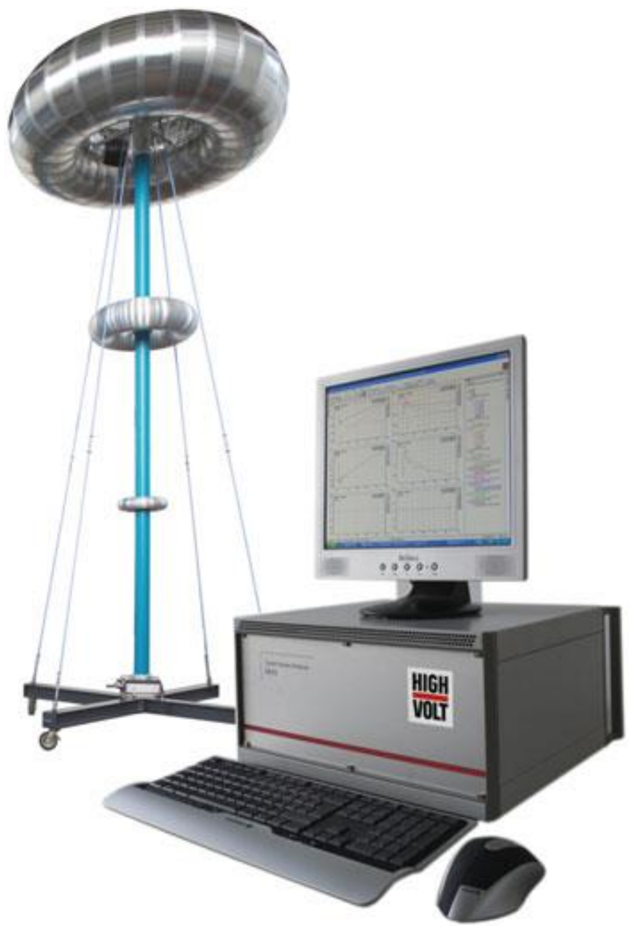
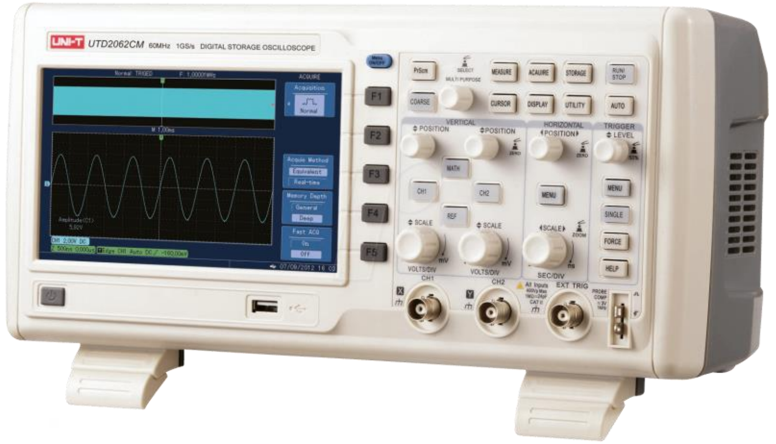


Figure (61) HV measuring system consisting of voltage divider, coaxial cable and PC-based digital recorder

Chapter 8
Cathode Ray Oscilloscope (CRO) and Digital Storage Oscilloscope (DSO)

9.1-Introduction: An oscilloscope, previously called an oscillograph, and informally known as a scope or o-scope, CRO (for cathode-ray oscilloscope) figure (62-a), or DSO (Digital Storage Oscilloscope) figure (62-b), is a type of electronic test instrument that allows observation of varying signal voltages, usually as a two-dimensional plot of one or more signals as a function of time. Other signals (such as sound or vibration) can be converted to voltages and displayed.



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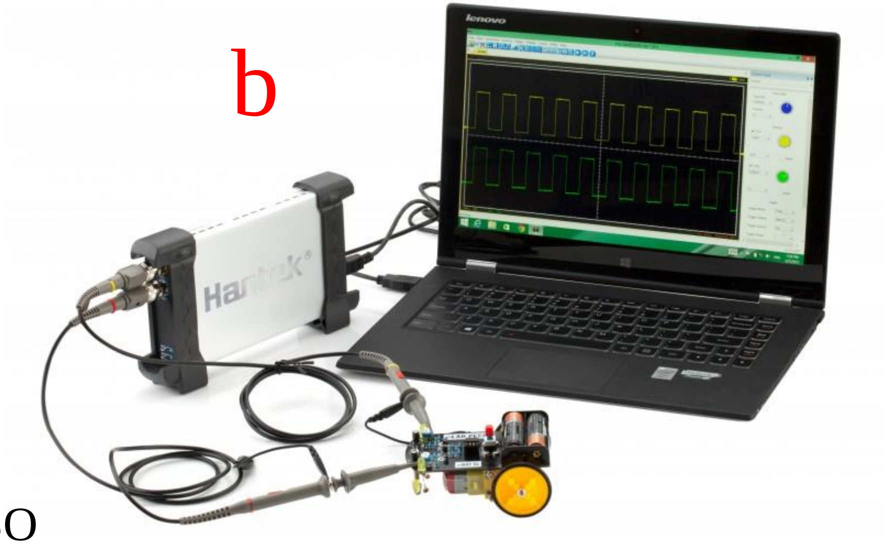


b

Figure (62) CRO and DSO

Engineers, scientists, and other technical professionals around the world depend on oscilloscopes as one of the primary voltage measuring instruments. This is an unusual situation because the oscilloscope is not the most accurate voltage measuring instrument usually available in the lab. It is the graphical nature of the oscilloscope that makes it so valued as a measurement instrument — not its measurement accuracy.

The output of a battery can be completely described by its output voltage and current. However, the output of a more complex signal source needs additional information such as frequency, pulse width, duty cycle, peak to peak, amplitude, overshoot, pre-shoot, rise time, fall time, ringing, oscillating and more to be completely described. Figure (63) shows what kind of details a waveform could have.

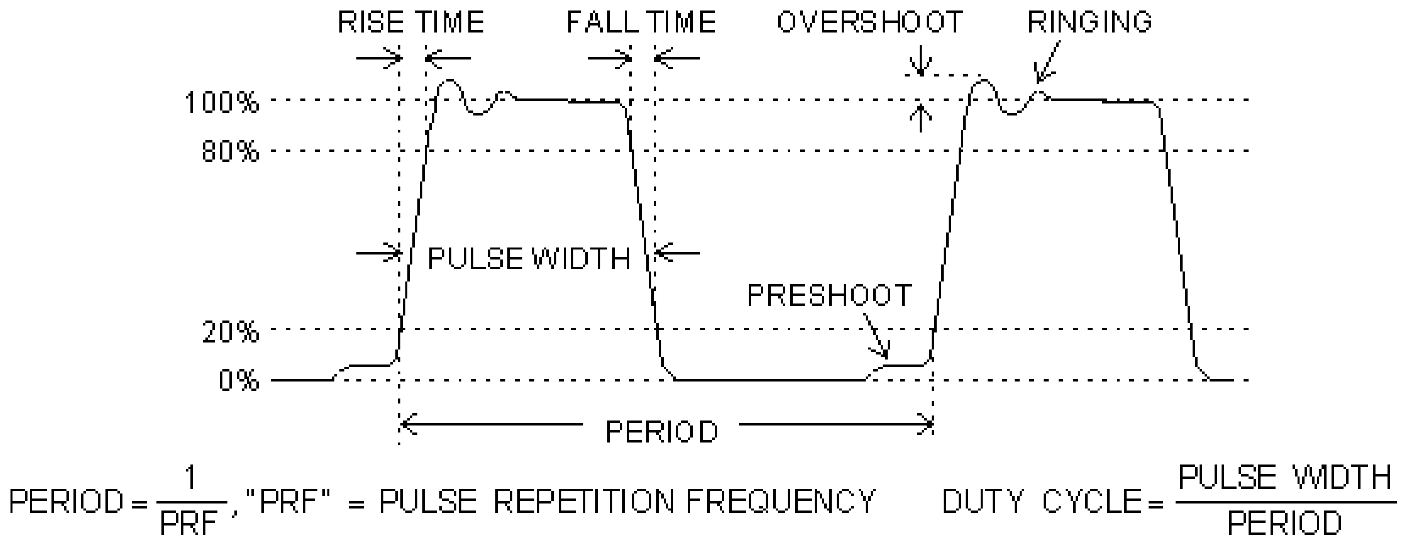


Figure (63) waveform characteristics

9.2- The Oscilloscope Block Diagram:

The oscilloscope contains four basic circuit blocks: the vertical amplifier, the time base, the trigger, and the display.

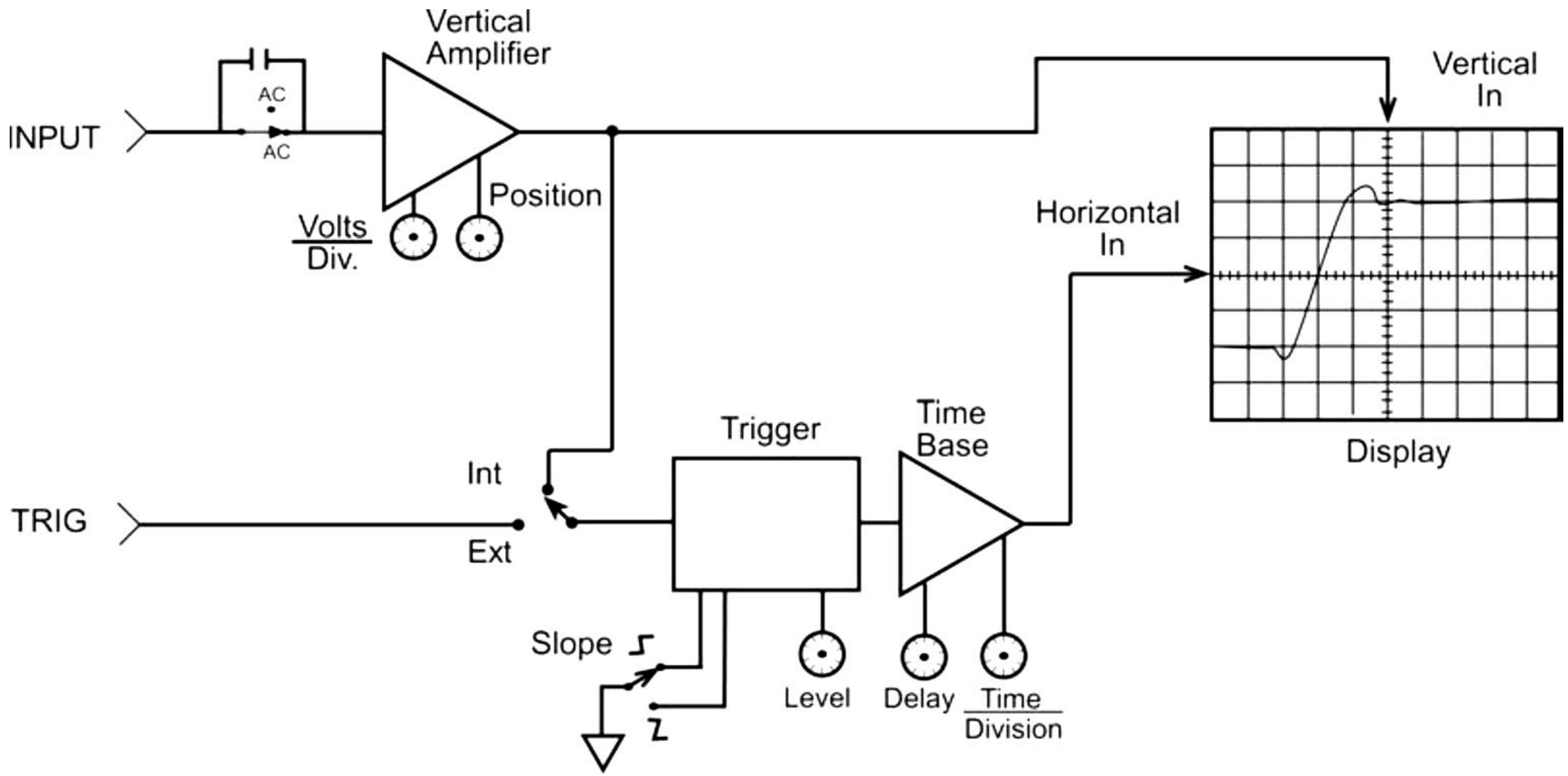
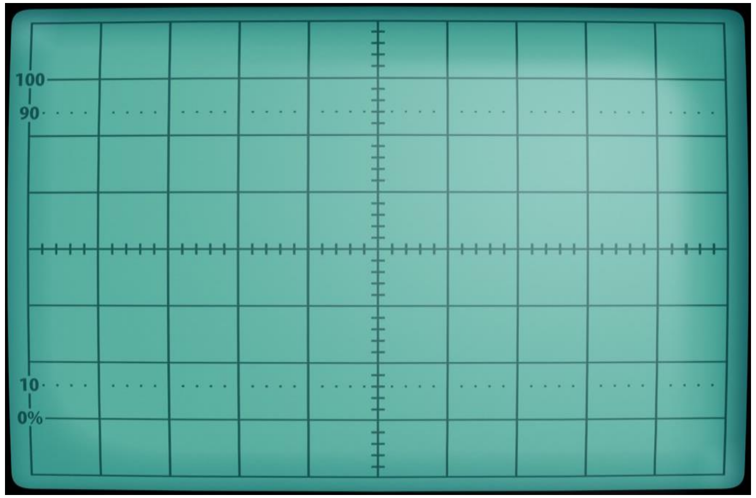


Figure (64) Oscilloscope Block Diagram

9.2.1- The display:

Of the four basic blocks of the oscilloscope, the most visible of these blocks is the display with its cathode-ray tube (CRT) shown in figure (65). This is the component in the oscilloscope that produces the graphical display of the input voltage and it is the component with which the user has the most contact.



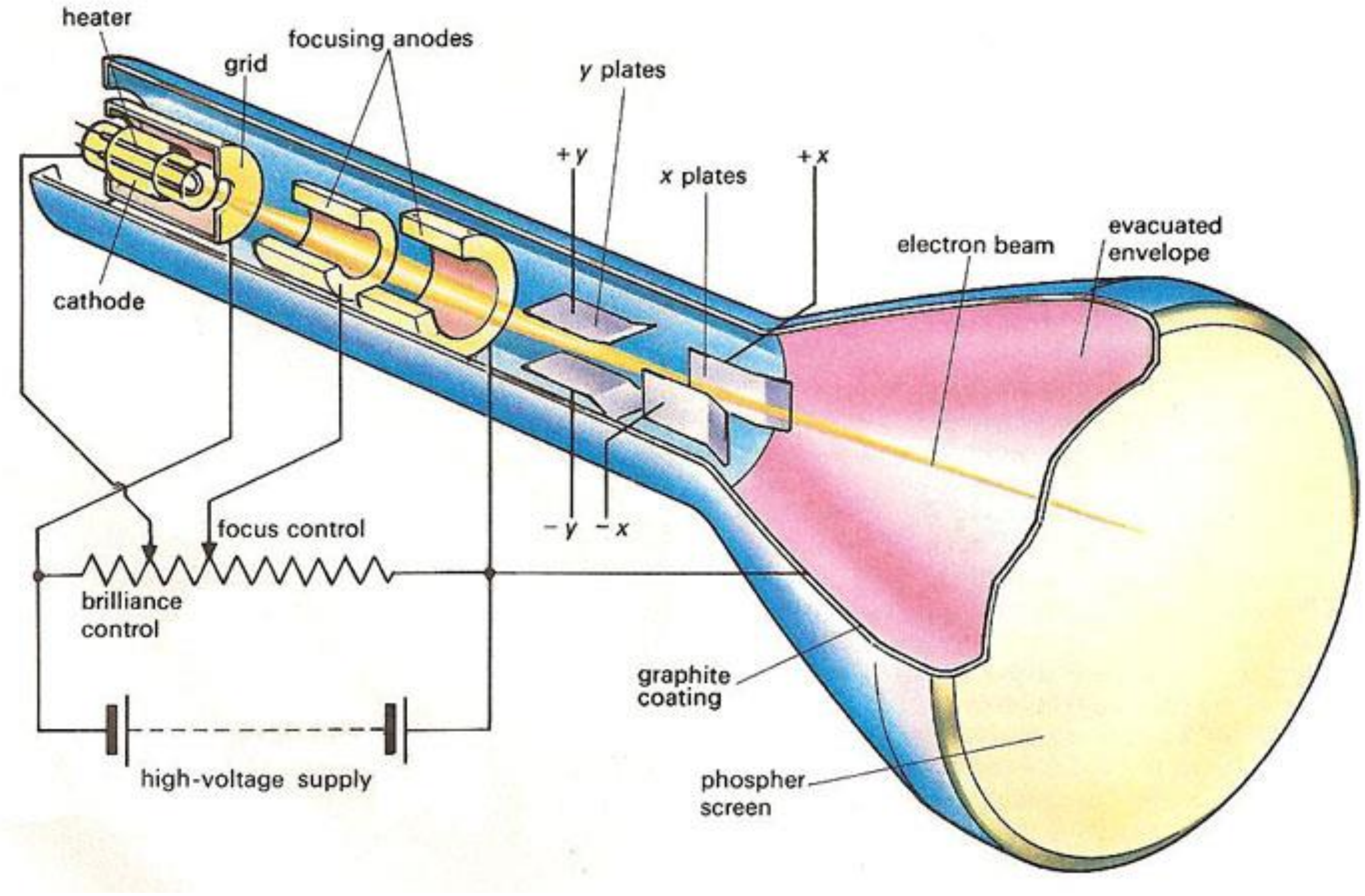


Figure (65) Display screen and cathode-ray tube

9.2.2- The vertical amplifier:

The vertical amplifier conditions the input signal so that it can be displayed on the CRT. The vertical amplifier provides controls of volts per division, position, and coupling, allowing the user to obtain the desired display. This amplifier must have a high enough bandwidth to ensure that all of the significant frequency components of the input signal reach the CRT.

In the case of the digital oscilloscope, the vertical amplifier block will include the ADC and high-speed waveform memory. For the Analog scope the vertical block will include delay lines with their associated drivers and a power amplifier to drive the CRT plates.

9.2.3- The trigger:

The trigger is responsible for starting the display at the same point on the input signal every time the display is refreshed.

9.2.4- Time base (Horizontal amplifier):

The time base is the part of the oscilloscope that causes the input signal to be displayed as a function of time. The circuitry in this block causes the CRT beam to be deflected from left to right as the input signal is being applied to the vertical deflection section of the CRT. Controls for time-per-division and position (or delay) allow the user of the oscilloscope to adjust the display for the most useful display of the input signal. The time-per-division controls of most oscilloscopes provide a wide range of values, ranging from a few nanoseconds to seconds per division.

9.3- The Oscilloscope function of knobs

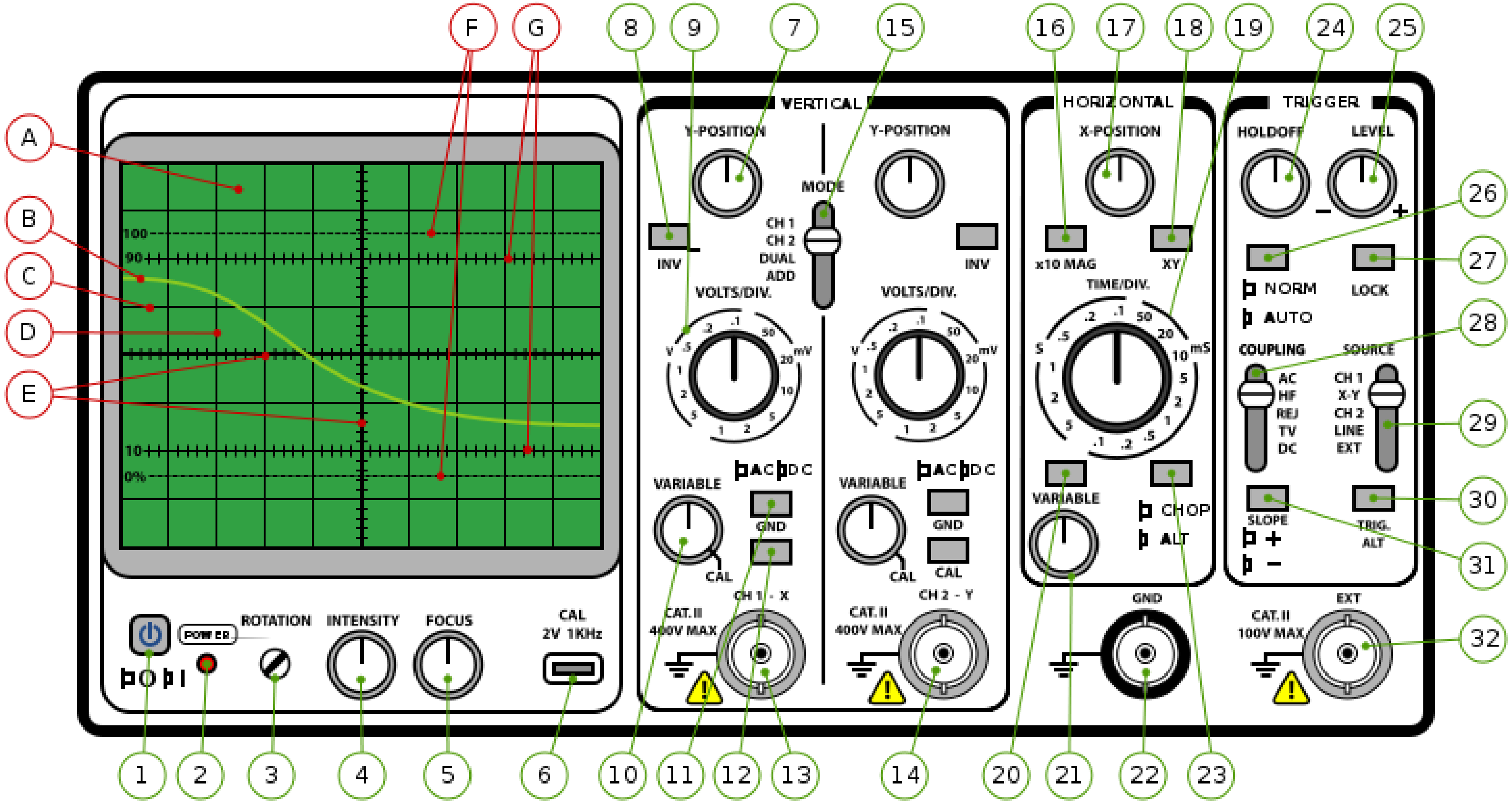


Figure (66) A front view diagram of a general Oscilloscope

9.3.1- Display:

A is the display. This can be a phosphor screen or an LCD, and is usually about 100 mm corner to corner.

B shows the 'trace'. This is the line drawn by the scope to represent the signal. On a CRO, this line is created by a bright dot moving across the screen at high speed. On a digital scope, the line is drawn on the LCD like a graphical calculator.

The screen is overlaid with a grid of horizontal (C) and vertical (D) lines, called the 'graticule', which divides the screen into squares, called 'major divisions'. The graticule is usually 10 major divisions wide and 8 tall.

The central horizontal and vertical lines (E) are usually thicker than the others and are divided into 'minor divisions', usually five per major division.

There are also special horizontal lines labelled "0" (2.5 divisions below the centre) and "100" (2.5 divisions above it). The "10" and "90" lines have tick marks like the central axes. These four horizontal lines are guides for scaling the signal for rise-time measurement. This will be discussed later.

9.3.2- Power, Calibration and Display Controls:

1 is the Power On/Off Button.
2 is the Power Indicator which lights when the oscilloscope is on. This may be an LED in newer scopes or a neon tube in older scopes.

3 is the trace rotation (TR) control. This sets the inclination of a flat signal relative to the graticule. This is usually a Trim-pot “a small potentiometer used to make small adjustments” and needs to be set using a flat-bladed screwdriver. Once set, this control should retain its position and will rarely need adjusting.

4 is the intensity of the trace. Turning this up increases the brightness of the trace, and turning it down makes it dimmer. An overly bright trace can damage the phosphor of the screen if the dot is moving too slowly.

5 is the focus control. The trace can get fuzzy if the electron beam is not focused correctly. The focus control (5) sets this. Most scopes can focus the beam to form a trace about 1mm wide.

6 is the calibration point. This gives a steady square wave at a set frequency and voltage, allowing the scaling of the trace to be set accurately. Sometimes, more than one frequency and voltage is available to give a more representative calibration. The standard calibration signal is between 0V and 2V at 1KHz.

9.3.3- Vertical Axis Controls:

7 controls the position of the trace. It can be adjusted to set the voltage relative to a ground, or it can be adjusted to separate the two signals - perhaps the first channel in the top half of the screen and the second channel in the bottom.

8 inverts the relevant channel. That is, the negative voltage is displayed, and the trace is upside-down.

9 is the vertical scale control, often called the volts/div. control. This sets the height of the trace. It operates in discrete steps. For example 5 volts per one major vertical division.

10 is a variable height control. It can adjust the height of the trace up to the next set increment on the volts/div. control. When set to CAL, the height is as stated on the volts/div. control.

11 is the AC/DC toggle. When set to AC, any DC component of the voltage is filtered out by switching a capacitor in series with the input signal, leaving just an AC voltage. This is useful when the DC component swamps the AC component, making it either too small to see or driving it off the top of the screen. When set to DC, the signal is displayed as it is.

12 is the GND toggle. By selecting this, the input signal is ignored, and the trace shows 0V. This can be useful to measure a voltage or to eliminate one of the traces from the display.

13 is the Channel 1 signal input. This is where the oscilloscope's probe is plugged in.
14 is the Channel 2 signal input. This is where the oscilloscope's probe is plugged in.
15 is the Mode control. Used to change the display mode into they following four types:
CH1 = Channel 1 only, CH2 = Channel 2 only, Dual = Both channels displayed on screen, Add = shows the sum of the two traces as one trace.

For example By inverting the traces, one can be subtracted from the other. This can be seen in the illustration figure (67) below. This shows a square wave on one channel and a sinusoidal wave on the other. On the left, the scope is set to "dual", and the two traces are shown side by side. On the right, the scope is set to "add", and the trace is the sum of the two signals.

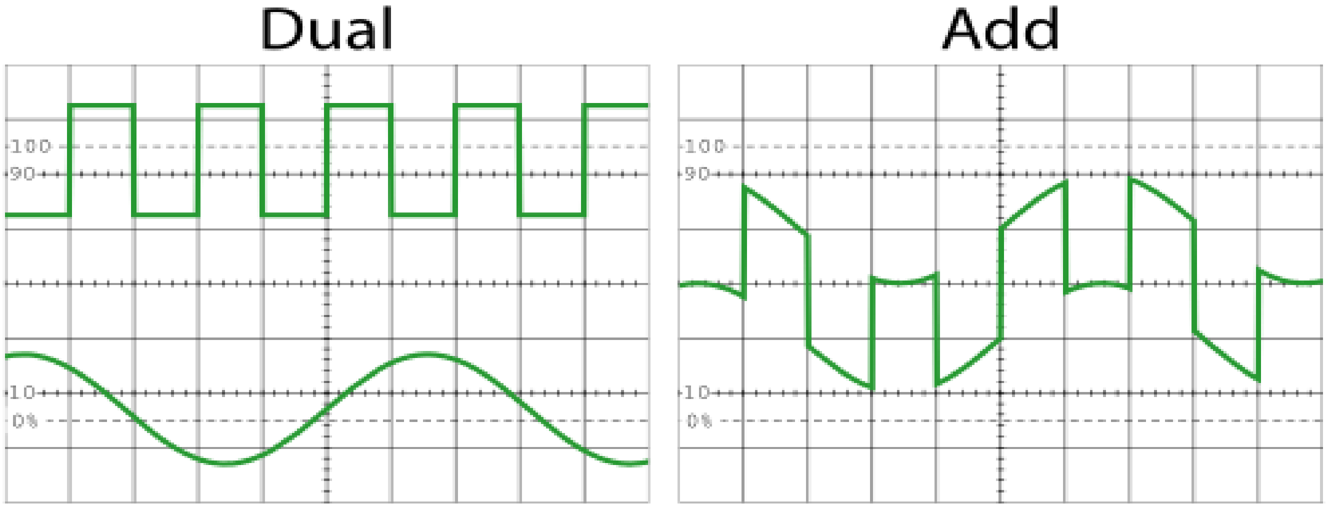


Figure (67) A front view diagram of a general Oscilloscope

9.3.3- Horizontal Axis Controls:

When operating in the normal voltage vs. time mode “Be aware of point number18”, this axis represents time. The primary control is the time base selector.

16 The ×10 MAG control, is a very useful control if you want to quickly zoom in on a feature without changing the time base and losing your settings. This button magnifies the central area of the trace by a factor of 10 in the horizontal direction (but leaves the voltage height unchanged).

17 The position of the trace from side to side is controlled by this x position knob. This is useful if part of the trace is off the edge of the screen but you don't want to change the time base.

18 XY toggles the mode between the usual voltage vs. time format and the XY mode. This continuously plots the voltage on Channel 1 along the horizontal axis against the voltage on Channel 2 (the vertical axis). This can be extremely useful to analyse frequency or phase relationships.

19 Time/Div. The time base is the length of time displayed per major horizontal division on the screen. This ranges from about 0.1 milliseconds to about 1 second (or more on digital scopes).

20 and 21 act in much the same way as point 10 does a variable width control on the horizontal axis. This diagram shows it to be slightly different from the vertical control.

To select a non-standard time base, press 20, and adjust 20 until the correct setting is obtained. To return to a calibrated time base, press 20 again. Sometimes these controls are the same style as 10.

22 is the GND terminal of the scope. This is used to set a "datum" voltage against which to measure the voltages on the input channels. Be careful when using isolated mains voltage circuits, as the "ground" is sometimes floating at mains voltage, and can short to the real ground, causing injury or death.

23 toggles between chop-mode and alt-mode. Chop-mode means that when the scope is drawing two signals side by side it alternates rapidly between the two over the course of passing across the screen. This action is called chopping. Alt-mode alternates at the end of each pass, and can appear to flicker at slow speeds.

9.3.4- Triggering Controls:

24 Hold off, provides continuous control of holdoff time between sweeps.

25 Level, to control the amplitude point on the trigger signal at which the sweep is triggered.

26 Norm, Auto, Auto-permits triggering on waveform having repetition of at least 20 Hz.
Norm-Sweep is initiated when an adequate trigger signal is applied. In the absence of a trigger signal, no baseline trace will be present.

27 Lock, used to lock the trigger parameters.

28 Coupling, Switch determines the method used to couple external signals to the trigger circuit, such as AC, DC or TV signals.

29 Source, Switch determines the source of the trigger signals that is coupled to the input of the trigger circuit. Could be CH1, CH2, Exit, or Line “signal from a sample of the ac power source waveform”

30 Trig, ALT. ALT from alternate, used to alternate the sweeping between CH1 and CH2, it displays both channels on the screen and the sweep cannot be noticed at high speed sweeping. Some OSC have Chop button used to make both channels sweep at the same time.

31 Slope, switch-selects the slope of the signal that triggers the sweep. Either triggered on the positive-going portion of the trigger signal. Or on the negative portion of the trigger signal.

32 EXT from external, Connector provides a means of introducing external signals into the trigger generator.

9.4- Analog or Digital:

The analog oscilloscope applies the input signal to the vertical deflection plates of the CRT where it causes the deflection of a beam of high-energy electrons moving toward the phosphor-coated faceplate. The electron beam generates a lighted spot where it strikes the phosphor. The intensity of the light is directly related to the density of the electrons hitting a given area of the phosphor. Because this analog operation is not based on any digitizing techniques, most people have little trouble creating a very accurate and simple mental model in their minds of its operation.

The digital oscilloscope or digital storage oscilloscope (DSO) differs from its analog counterpart in that the input signal is converted to digital data and therefore it can be managed by an embedded microprocessor. The waveform data can have correction factors applied to remove errors in the scope’s acquisition system and can then be stored, measured, and/or displayed. That the input signal is converted from analog to digital and manipulations are performed on it by a microprocessor results in people not having a good mental model of the digital oscilloscope’s operation.

A Comparison of Analog and Digital Oscilloscopes		
	Analog Oscilloscope	**Digital Oscilloscope**
Operation	Simple	Complex
Front panel controls	Direct access knobs	Knobs and menus
Display	Real-time vector	Digital raster scan
Gray scales	>16	>4
Horizontal resolution	>1000 lines	500 lines
Dead-time	Short	Can be long
Aliasing	NO	Yes
Voltage accuracy	±3% of full scale	±3% of full scale
Timing accuracy	±3% of full scale	±0.01% of full scale
Single shot capture	None	Yes
Glitch capture	Limited	Yes
Waveform storage	None	Yes
Pre-trigger viewing	None	Yes
Data out to a computer	NO	Yes

9.5- Testing the Probe

Let’s connect that channel up to a meaningful signal. Most scopes will have a built-in frequency generator that emits a reliable, set-frequency wave – on the GA1102CAL there is a 1kHz square wave output at the bottom-right of the front panel. The frequency generator output has two separate conductors – one for the signal and one for ground. Connect your probe’s ground clip to the ground, and the probe tip to the signal output as shown in figure (68).

Figure (68) Testing the Probe

As soon as you connect both parts of the probe, you should see a signal begin to dance around your screen. Try fiddling with the horizontal and vertical system knobs to manoeuvre the waveform around the screen. Rotating the scale knobs clockwise will "zoom into" your waveform, and counter-clockwise zooms out. You can also use the position knob to further locate your waveform.

If your wave is still unstable, try rotating the trigger position knob. Make sure the trigger isn't higher than the tallest peak of your waveform.

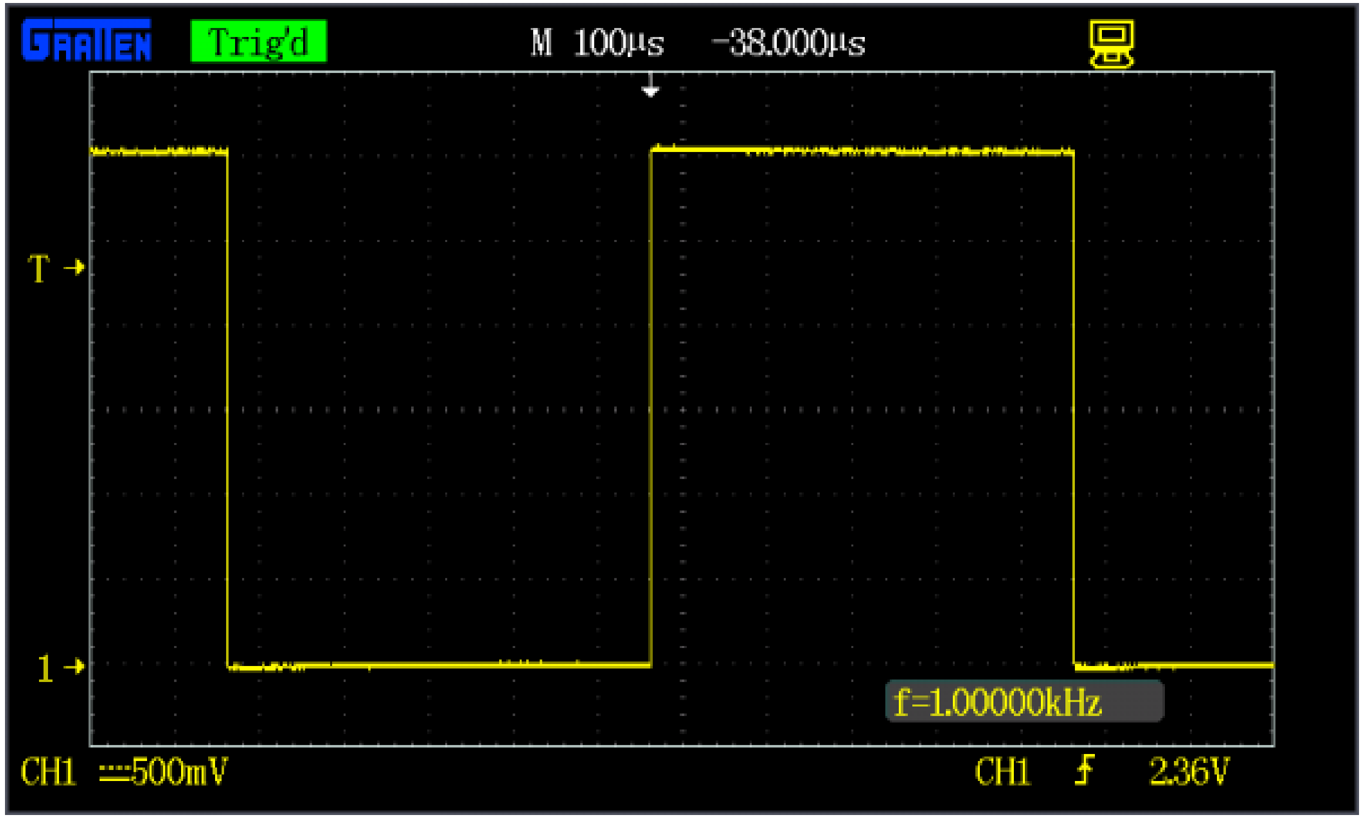
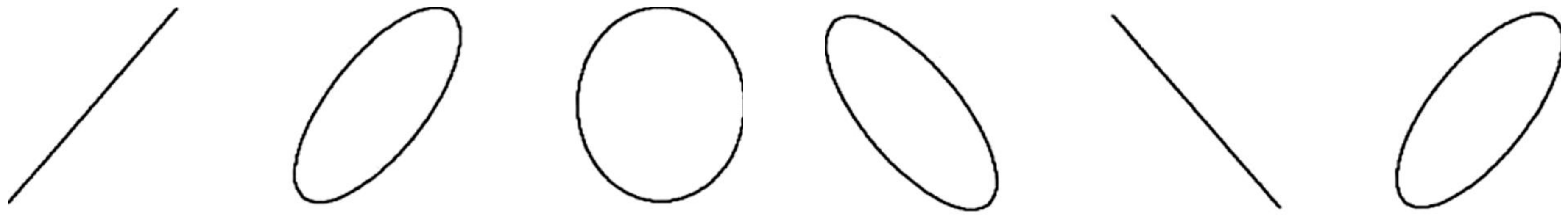


Figure (69) Testing the Probe

By default, the trigger type should be set to edge, which is usually a good choice for square waves like this. Try fiddling with those knobs enough to display a single period of your wave on the screen. Or try zooming way out on the time scale to show dozens of squares.

Chapter 9
Lissajous patterns

Lissajous patterns are sometimes used for the measurement of phase. They are produced in an oscilloscope by connecting one signal to the vertical trace and the other to the horizontal trace. If the ratio of the first frequency to the second is a rational number (i.e., it is equal to one small integer divided by another), then a closed curve will be observed on the CRO.



$(a)0$ $\quad$ $(\mathrm{b})\frac{\pi}{4}$ $\quad$ $(\mathrm{c})\frac{\pi}{2}$ $\quad$ $(\mathrm{d})\frac{3\pi}{4}$ $\quad$ $(\mathrm{e})\ \pi$ $\quad$ $(\mathrm{f})\frac{-\pi}{4}$

Figure (70) Lissajous figures for two equal-amplitude, frequency synchronized signals with a relative phase difference of $(a)0$, $(\mathrm{b})\frac{\pi}{4}$, $(\mathrm{c})\frac{\pi}{2}$, $(\mathrm{d})\frac{3\pi}{4}$, $(\mathrm{e})\ \pi$, $(\mathrm{f})\frac{-\pi}{4}$

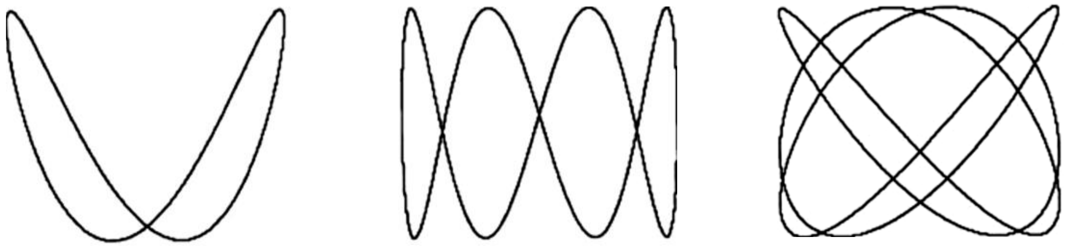


Figure (71) Lissajous figures for two signals with vertical frequency: horizontal frequency ratios of (a) 2:1 , (b) 4:1 , (C) 4:3

Figure (72) Lissajous figures for two signals with synchronized frequency and various phase differences:
(a) phase difference = 0°
(b) phase difference = 45°
(c) phase difference = 90°
(d) phase difference = 135°
(e) phase difference = 180°
(f) phase difference = 315°

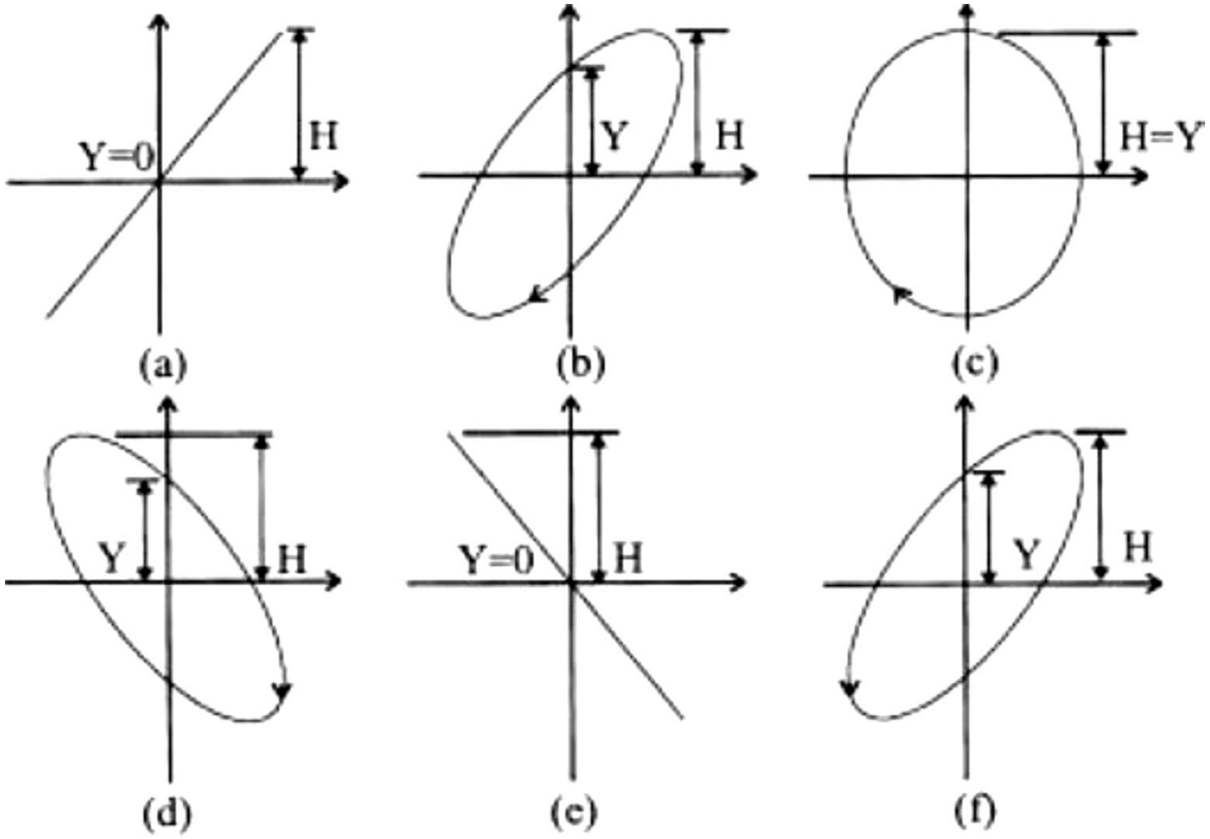


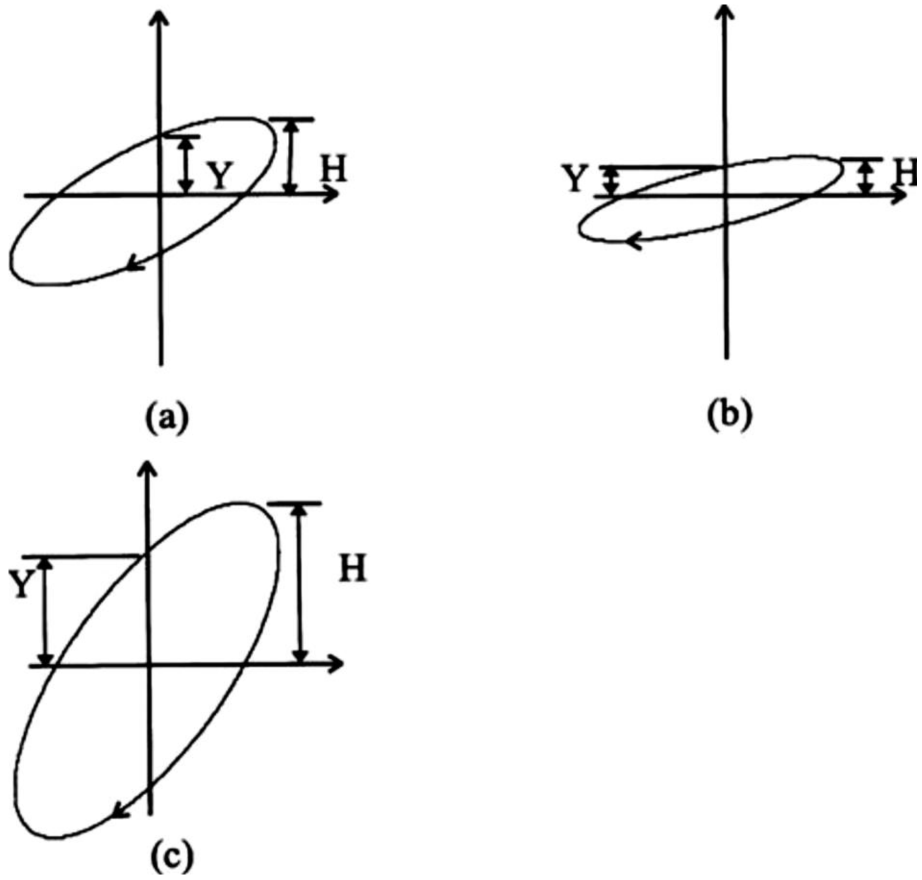
Figure (72) Lissajous figures for two signals with synchronized frequency and various phase differences:

If the two frequencies are unrelated, then there will be only a patch of light observed because of the persistence of the oscilloscope screen. If the two signals have the same frequency, then the Lissajous figure will assume the shape of an ellipse. The ellipse's shape will vary according to the phase difference between the two signals, and according to the ratio of the amplitudes of the two signals. Figure 5.6 shows some figures for two signals with synchronized frequency and equal amplitudes, but different phase relationships. The formula used for determining the phase is:

$$Sin\ (\varphi) = \pm \frac{Y}{H}$$

Where *H* is half the maximum vertical height of the ellipse and *Y* is the intercept on the *y-axis*.
Figure (73) shows some figures for two signals that are identical in frequency and have a phase difference of 45°, but with different amplitude ratios. Note that it is necessary to know the direction that the Lissajous trace is moving in order to determine the sign of the phase difference. In practice, if this is not known a priori, then it can be determined by testing with a variable frequency signal generator. In this case, one of the signals under consideration is replaced with the variable frequency signal. The signal generator is adjusted until its frequency and phase equal that of the other signal input to the CRO. When this happens, a straight line will exist. The signal generator frequency is then increased a little, with the relative phase thus being effectively changed in a known direction.

Figure (73) Lissajous figures for two signals with synchronized frequency, a phase difference of 45°, and various amplitude ratios: (a) amplitude ratio of 1, (b) amplitude ratio of 0.5, (c) amplitude ratio of 2.



Lissajous figure methods are a little more robust to noise than direct oscilloscope methods. This is because there are no triggering problems due to random noise fluctuations. Direct methods are, however, much easier to interpret when harmonics are present. The accuracy of oscilloscope methods is comparatively poor. The uncertainty of the measurement is typically in excess of 1°.