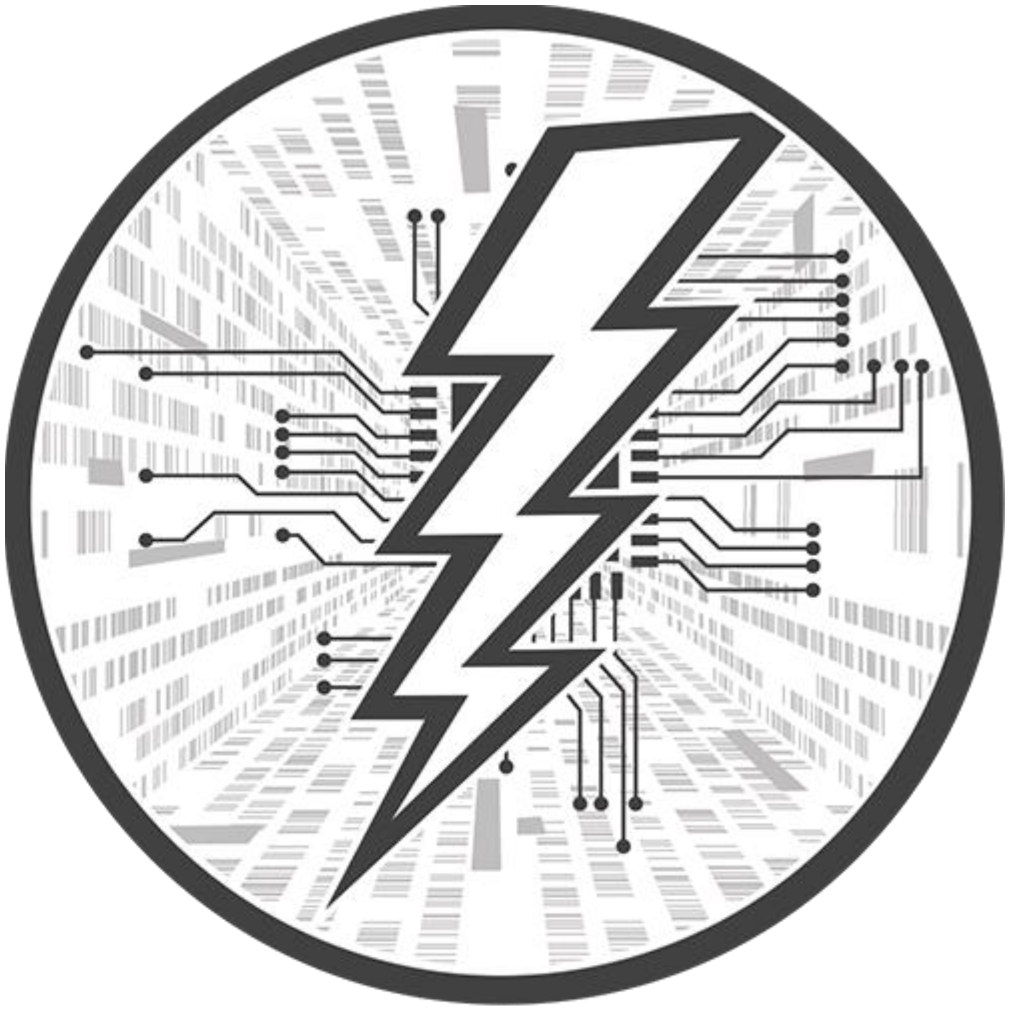
POWER ELECTRONICS



By: MSc Eng. ISMAEIL ALNAAB

Third stage, 2017 – 2018

- Research has shown that students understand more from a lecture if they write things down. Keep focus and ask if you did not understand.

- Study to Learn …… Not just to pass an exam.

- Your attendance is important to you, not for me. Make sure you sign the attendance sheet.

- Homework's are crucial, 10% of your mark depends on them.

- This course has a computer class, where simulation of converter and inverter systems are carried out.

- Do not fully depend on me, go and search the YOUTUBE and GOOGLE.

- Try to understand why are you studying this heavy material, is there any relationship between this material and the real world.

Syllabus :

Chapter 1: Power Devices and Switch elements
Chapter 2: Thyristor family and Thyristor ratings
Chapter 3: AC to DC converters
Chapter 4: DC to DC converters
Chapter 5: AC converters
Chapter 6: DC to AC converters
Chapter 7: UPS (Uninterruptible power supply)

Introduction :

Power electronics involves the study of electronic circuits intended to control the flow of electrical energy. These circuits handle power flow at levels much higher than the individual device ratings. The control of electric motor drives requires control of electric power. Power electronics have eased the concept of power control. Power electronics signifies the word power electronics and control or we can say the electronic that deal with power equipment for power control. Power electronics represents a median point at which the topics of energy systems, electronics, and control converge and combine as shown in figure (1).

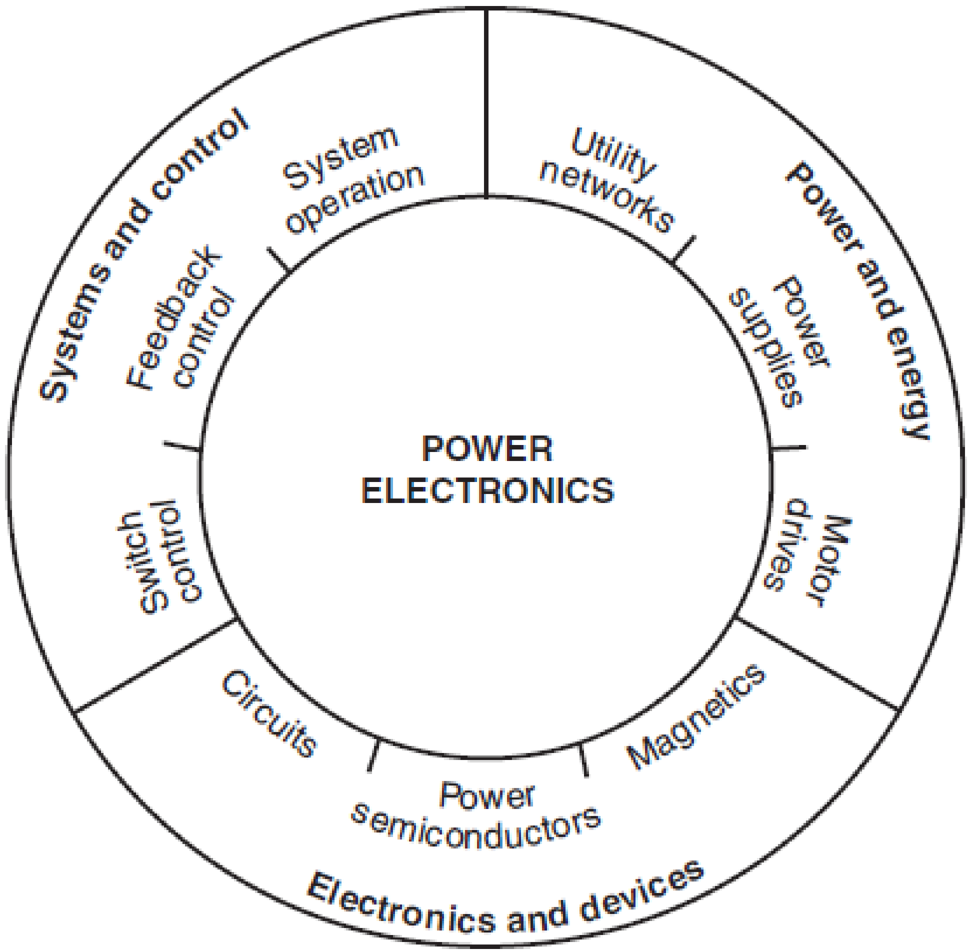


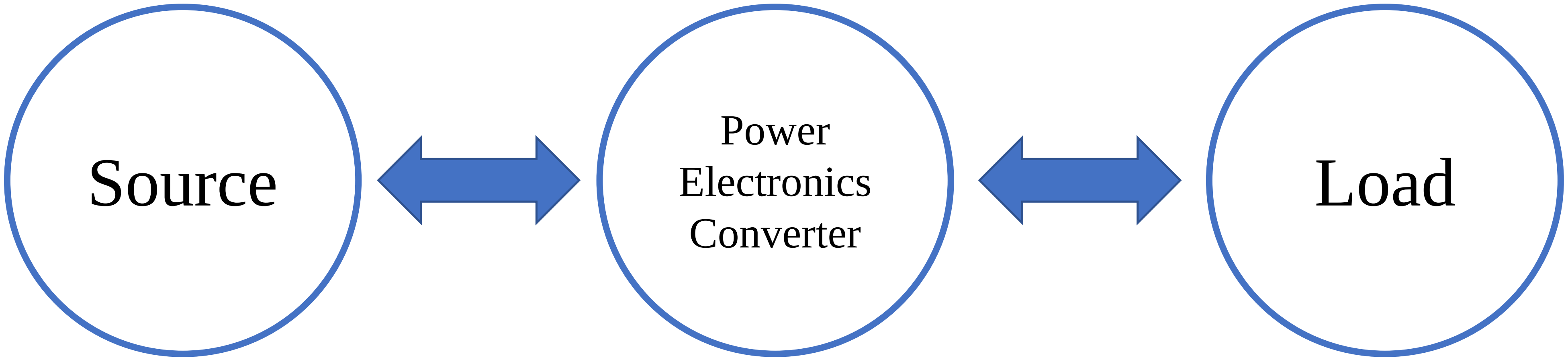
Figure (1) Control, energy, and power electronics are interrelated

The design of power conversion equipment includes many disciplines from electrical engineering. Power electronics includes applications of circuit theory, control theory, electronics, electromagnetics, microprocessors (for control), and heat transfer. Advances in semiconductor switching capability combined with the desire to improve the efficiency and performance of electrical devices have made power electronics an important and fast-growing area in electrical engineering.

Power electronics based on the switching of power semiconductor devices. With the development of power semiconductor technology, the power handling capabilities and switching speed of power devices have been improved tremendously.

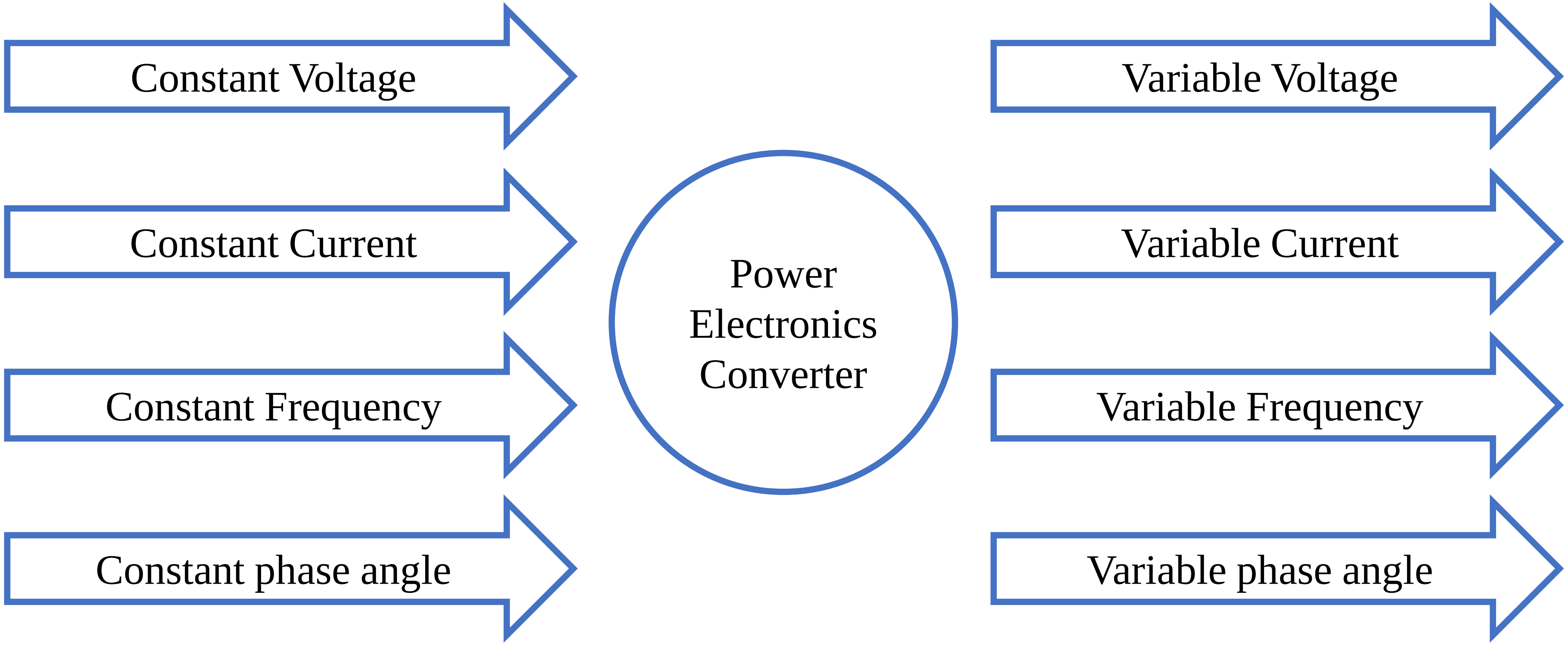
Applications of power electronics:

Power supplies, Hand power tools, White goods, Automotive, Railways, Aerospace, Renewable energy, Energy transmission, Power plants, Welding, Induction heating, Lighting, Telecommunications.

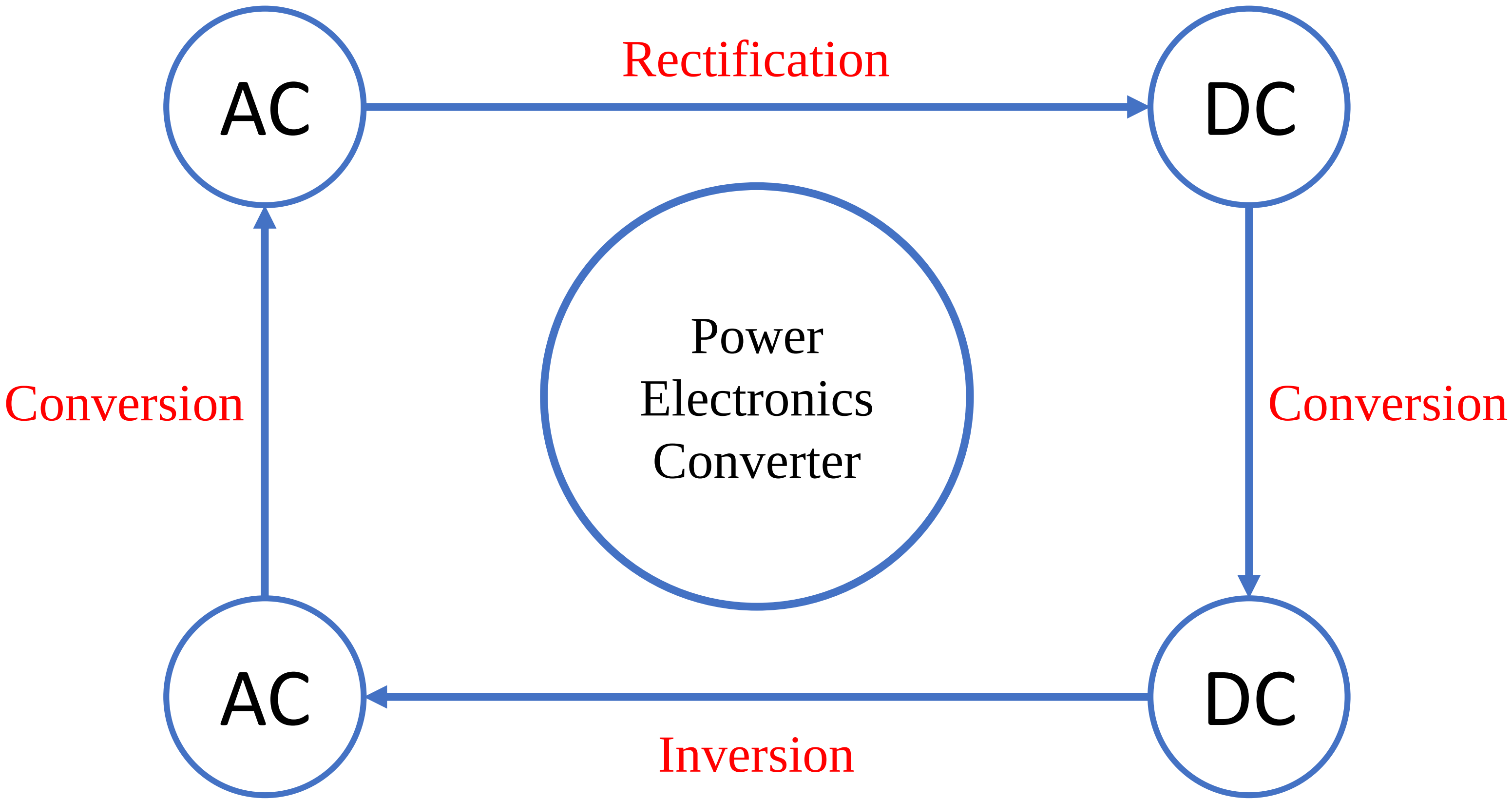


Electric grid
Battery
Solar panel
Wind power generator

Electric motor
Microprocessor
Electric welding
Lighting

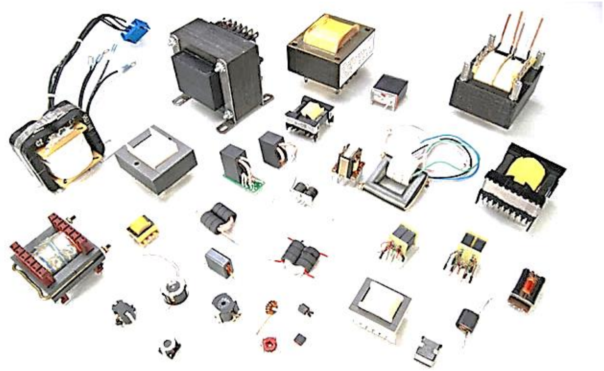

Constant Voltage
Constant Current
Constant Frequency
Constant phase angle
Power Electronics Converter
Variable Voltage
Variable Current
Variable Frequency
Variable phase angle

What does a Power Electronics Converter convert? $\left.\begin{array}{l}\text{Voltage}\\ \text{Current}\end{array}\right\}$ Magnitude, Frequency, Phase angle

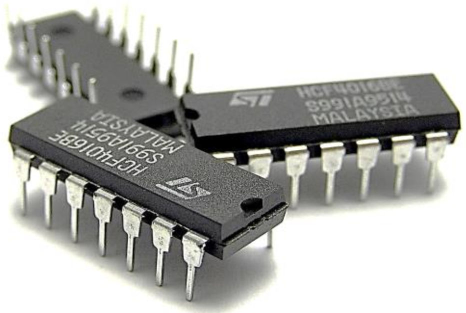


What are the Components for Power Electronic Converters?

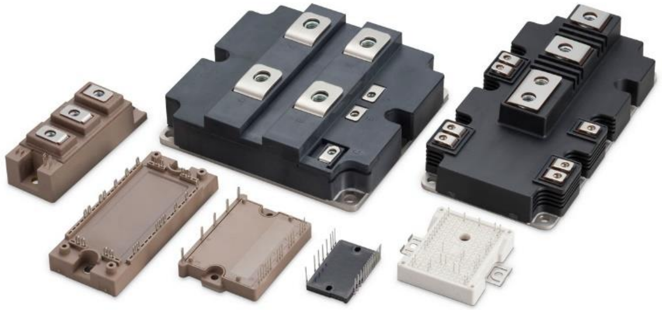
Capacitors



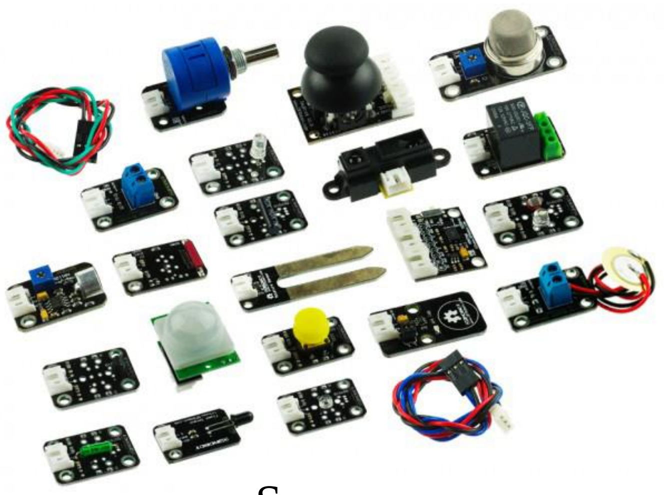
Transformers



Controllers



Power semiconductor switches



Sensors

Inductors

Resistors

Chapter 1
Power Devices and Switch elements

By turning the switch on and off with a variable duty ratio, we can control the average power delivered to the load, as shown in figure (2).

The first SCR was developed in late 1957. Power semiconductor devices are broadly categorized into 3 types:

1. Power diodes (600V,4500A)
2. Thyristors (10KV,300A,30MW)
3. Transistors

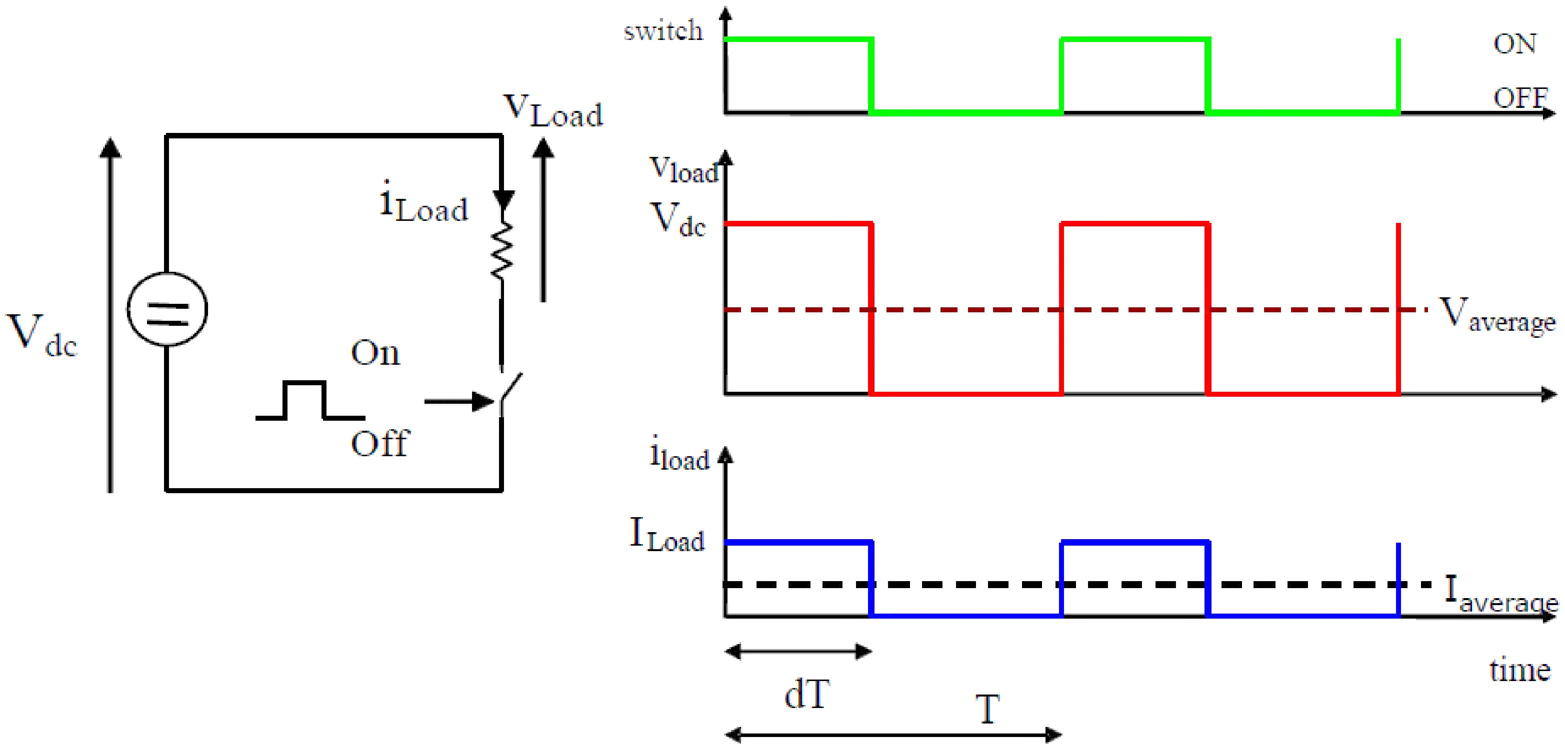


Figure (2) how a switch can control the average output current and voltage

1- Power Diodes

A diode is the simplest electronic switch. It is uncontrollable, in that the on and off conditions are determined by voltages and currents in the circuit. The diode is forward-biased (on) when the current id Figure (3-a) is positive and reverse-biased (off) when vd is negative. In the ideal case, the diode is a short circuit when it is forward-biased and is an open circuit when reverse-biased.

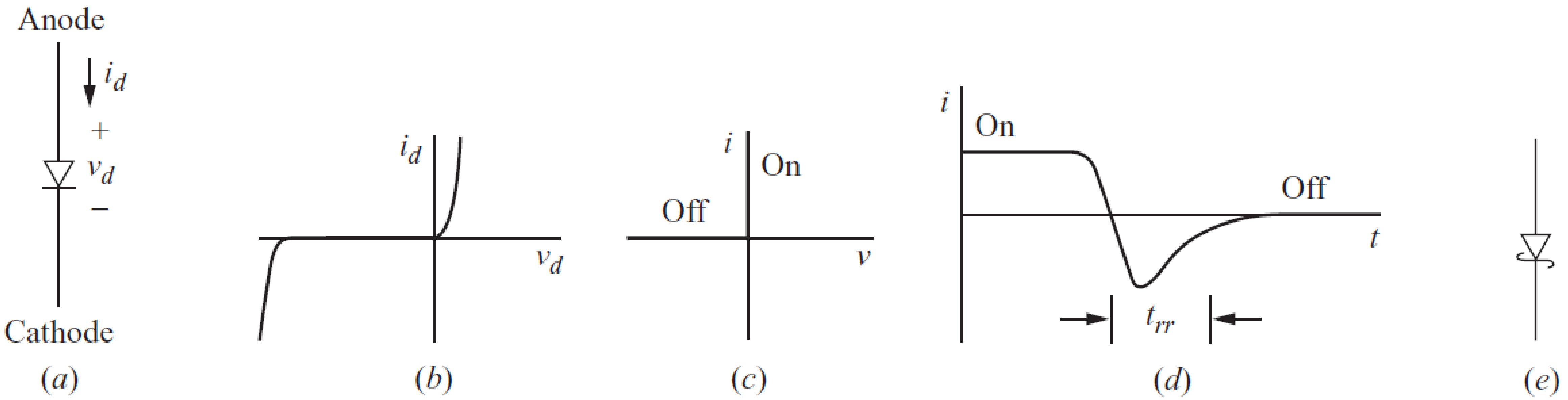


Figure (3) (*a*) Rectifier diode; (*b*) *i-v* characteristic; (c) idealized *i-v* characteristic; (*d*) reverse recovery time *trr*;
(*e*) Schottky diode.

The actual and idealized current-voltage characteristics are shown in Figure (3-b and c). The idealized characteristic is used in most analyses in this material. An important dynamic characteristic of a non-ideal diode is reverse recovery current. When a diode turns off, the current in it decreases and momentarily becomes negative before becoming zero, as shown in Figure (3-d). The time trr is the reverse recovery time, which is usually less than 1 s. This phenomenon may become important in high-frequency applications. Fast-recovery diodes are designed to have a smaller trr than diodes designed for line-frequency applications. Silicon carbide (SiC) diodes have very little reverse recovery, resulting in more efficient circuits, especially in high-power applications. Schottky diodes Figure (3-e) have a metal-to-silicon barrier rather than a P-N junction. Schottky diodes have a forward voltage drop of typically 0.3 V. These are often used in low-voltage applications where diode drops are significant relative to other circuit voltages. The reverse voltage for a Schottky diode is limited to about 100 V. The metal-silicon barrier in a Schottky diode is not subject to recovery transients and turn-on and off faster than P-N junction diodes.

Types of Power Diodes:

A- Bipolar PN Junction
B- Schottky Diode
C- PIN Diode (I=Intrinsic)
D- Other types: Diode Bridge, Tunnel Diode, Varicap Diode, Zener Diode, LED

A- Bipolar PN Junction

Bipolar PN Junction diode circuit and model are shown in figure (4).
The diode i-v characteristics are shown in figure (5).

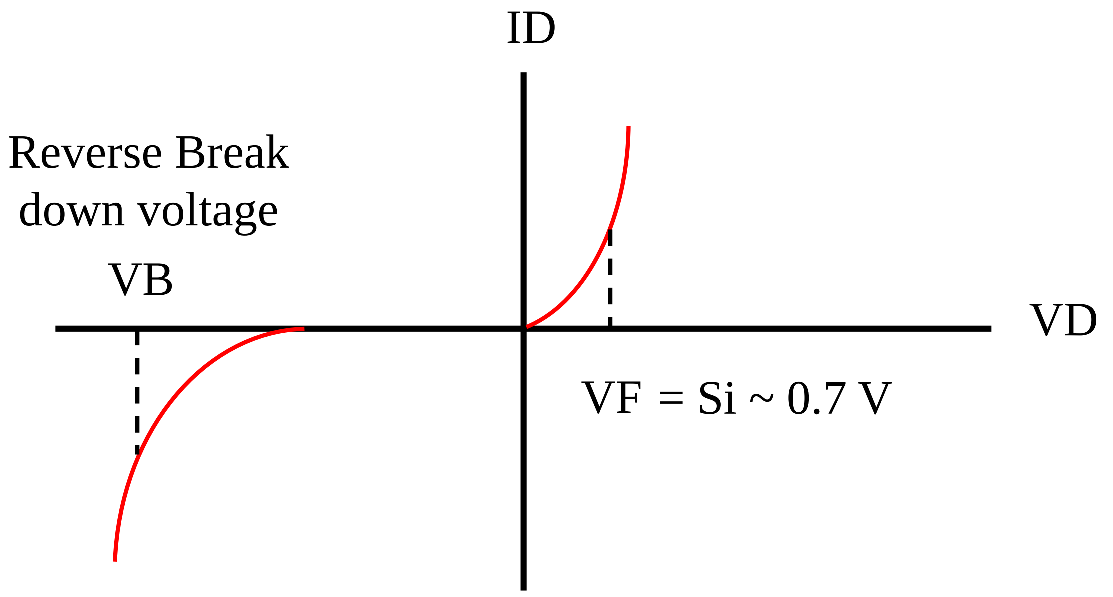


Figure (5) diode i-v characteristics

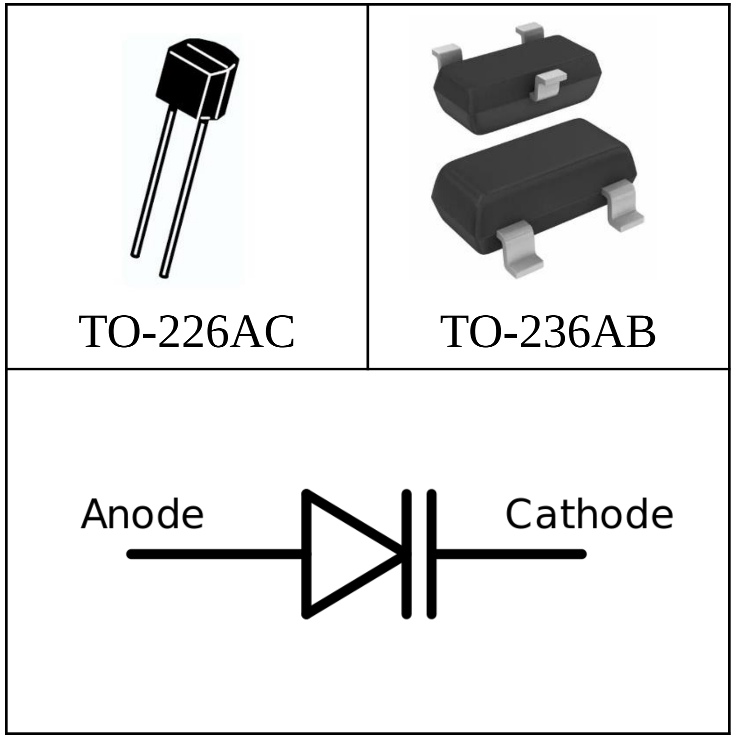


Figure (4) Bipolar PN Junction diodes

B- Power Schottky Diode

Power Schottky diode circuit and model are shown in figure (6). The Power Schottky diode i-v characteristics are shown in figure (7). A Schottky diode is formed by placing a thin film of metal in direct contact with a semiconductor. The i-v characteristic is similar to a pn diode although the fundamental physics are different than that of a pn junction. The forward voltage of a Schottky diode is lower than of a pn junction diode (0.3V-0.4V). Also the reverse voltage break down is lower, whereas the reverse current density is higher.

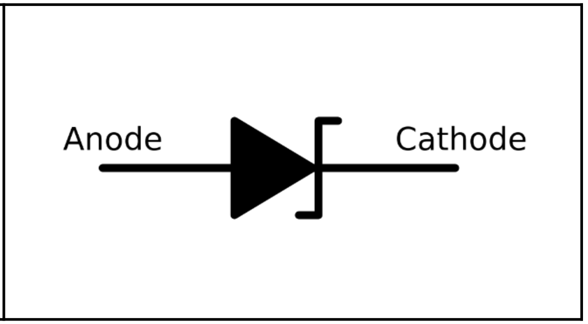


Figure (6) Schottky diode circuit and model

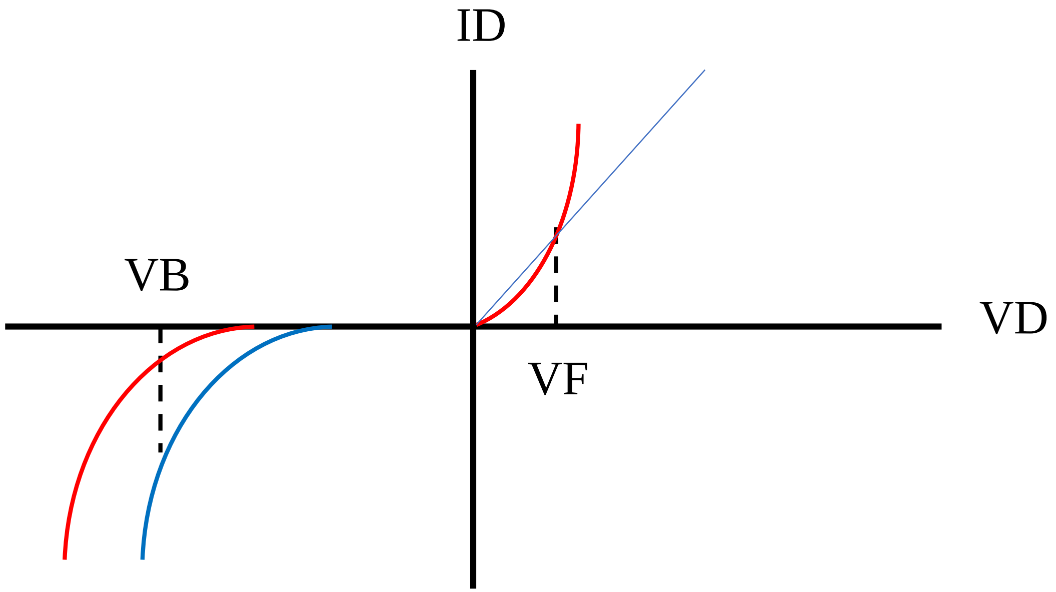


Figure (7) diode i-v characteristics

C- PIN Diode (I=Intrinsic)

A Power Diode is often termed PIN Diode (I=Intrinsic).
The circuit and model of PIN diode are shown in figure (8).
ld = drift region or intrinsic region
ld is a function of breakdown voltage
Power Diode Transient, reverse recovery i-v characteristics are shown in figure (9) where:
Vrr = reverse recovery voltage, Irr = reverse recovery current

$$Eoff = 0.5\ Qrr\ Vdc$$
$$Qrr = 0.5\ Irr\ Trr$$

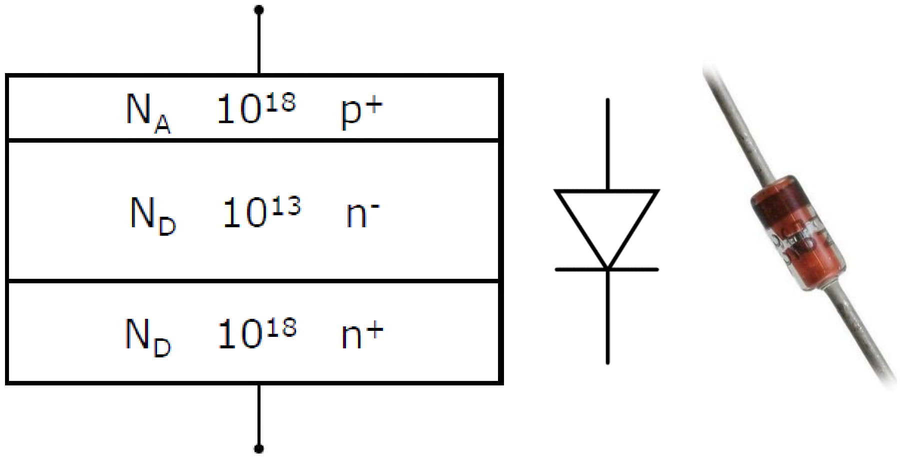


Figure (8) PIN diode circuit and model

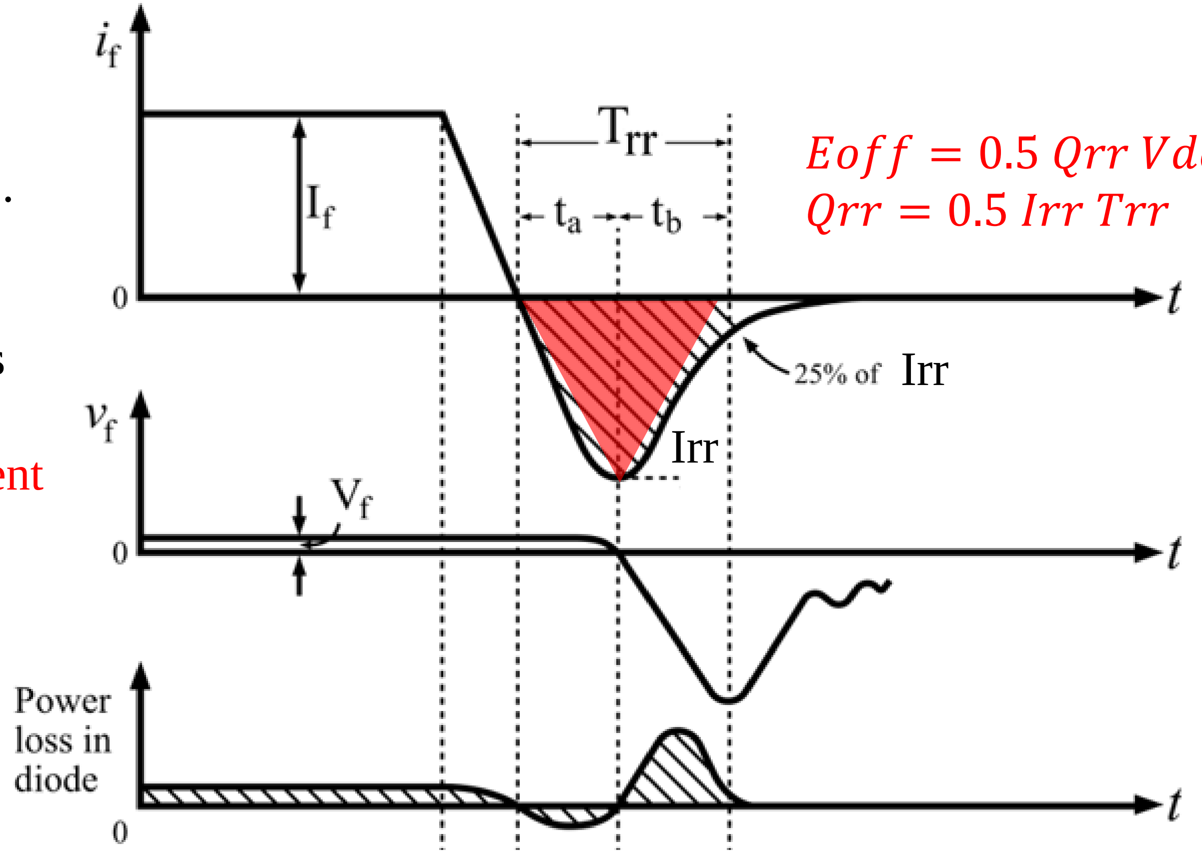
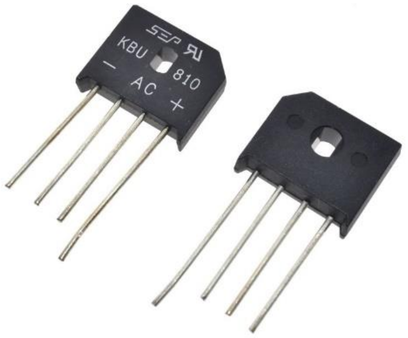
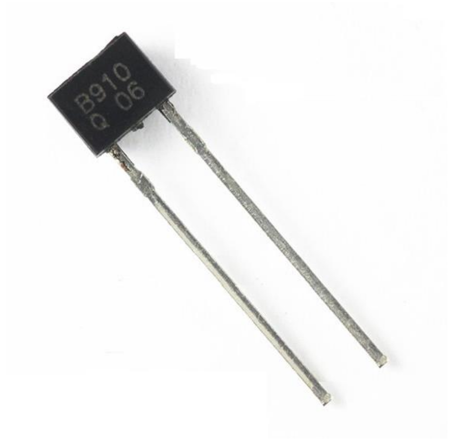


Figure (9) Power Diode Transient, reverse recovery i-v characteristics

D- Other types:

				
Diode Bridge	Tunnel Diode	Varicap Diode	Zener Diode	LED

Chapter 2
Thyristor family and Thyristor ratings

Electronic switches:

An electronic switch is characterized by having the two states on and off, ideally being either a short circuit or an open circuit. Applications using switching devices are desirable because of the relatively small power loss in the device. If the switch is ideal, either the switch voltage or the switch current is zero, making the power absorbed by it zero. Real devices absorb some power when in the on state and when making transitions between the on and off states, but circuit efficiencies can still be quite high. Some electronic devices such as transistors can also operate in the active range where both voltage and current are nonzero, but it is desirable to use these devices as switches when processing power. The particular switching device used in a power electronics circuit depends on the existing state of device technology. The behaviours of power electronics circuits are often not affected significantly by the actual device used for switching, particularly if voltage drops across a conducting switch are small compared to other circuit voltages. Therefore, semiconductor devices are usually modelled as ideal switches so that circuit behaviour can be emphasized. Switches are modelled as short circuits when on and open circuits when off. Transitions between states are usually assumed to be instantaneous, but the effects of non-ideal switching are discussed where appropriate.

Thyristors are electronic switches used in some power electronic circuits where control of switch turn-on is required. The term *thyristor* often refers to a family of three-terminal devices that includes the silicon-controlled rectifier (SCR), the triac, Bipolar Junction transistor (BJT), Insulated-gate bipolar transistor (IGBT), the gate turnoff thyristor (GTO), the MOS-controlled thyristor (MCT), and others. *Thyristor* and *SCR* are terms that are sometimes used synonymously. Thyristors are capable of large currents and large blocking voltages for use in high power applications, but switching frequencies cannot be as high as when using other devices such as MOSFETs. The three terminals of the SCR are the anode, cathode, and gate Figure (10-a). For the SCR to begin to conduct, it must have a gate current applied while it has a positive anode-to-cathode voltage. After conduction is established, the gate signal is no longer required to maintain anode current. The SCR will continue to conduct as long as the anode current remains positive and above a minimum value called the holding level. Figure (10-b) show the SCR circuit symbol and the idealized current-voltage characteristic. The gate turnoff thyristor (GTO) of Figure (10-c), like the SCR, is turned on by a short-duration gate current if the anode-to-cathode voltage is positive. However, unlike the SCR, the GTO can be turned off with a negative gate current. The GTO is therefore suitable for some applications where control of both turn-on and turnoff of a switch is required. The negative gate turnoff current can be of brief duration (a few microseconds), but its magnitude must be very large compared to the turn-on current. The idealized *i*-*v* characteristic is like that of Figure (10-b) for the SCR.

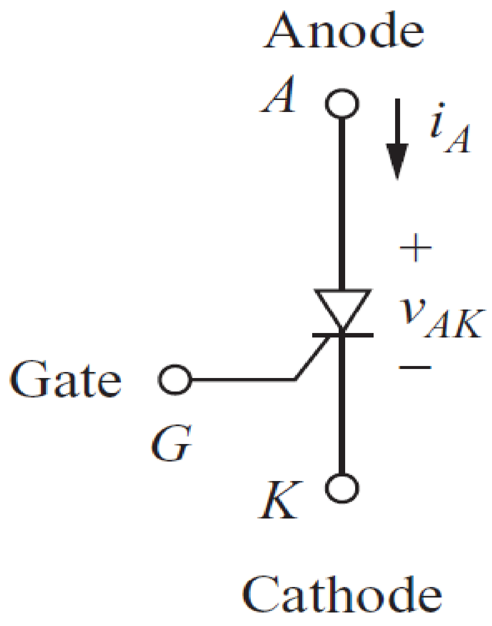


Figure (10-a)
Silicon-controlled
rectifier (SCR)

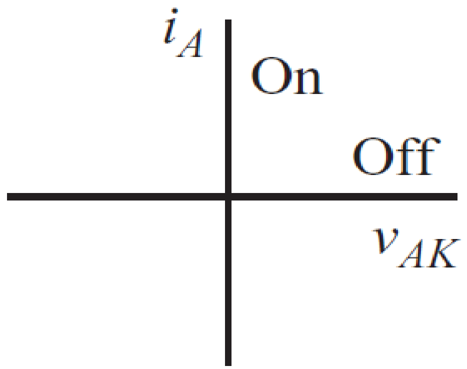


Figure (10-b)
SCR idealized *i*-*v*
characteristic

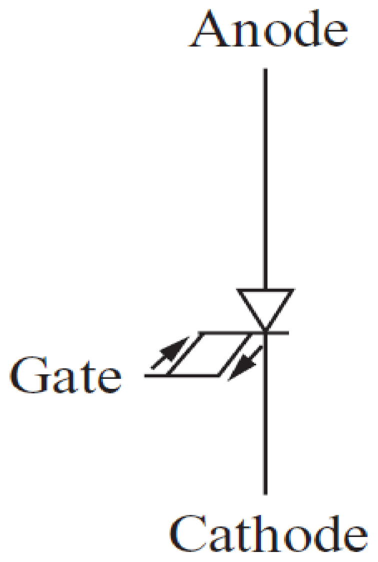


Figure (10-c)
Gate turnoff
(GTO) thyristor

The triac Figure (11-d) is a thyristor that is capable of conducting current in either direction. The triac is functionally equivalent to two antiparallel SCRs (in parallel but in opposite directions). Common incandescent light-dimmer circuits use a triac to modify both the positive and negative half cycles of the input sine wave. The MOS-controlled thyristor (MCT) in Figure (11-e) is a device functionally equivalent to the GTO but without the high turnoff gate current requirement. The MCT has an SCR and two MOSFETs integrated into one device. One MOSFET turns the SCR on, and one MOSFET turns the SCR off. The MCT is turned on and off by establishing the proper voltage from gate to cathode, as opposed to establishing a gate current in the GTO. Thyristors were historically the power electronics switch of choice because of high voltage and current ratings available. Thyristors are still used, especially in high-power applications, but ratings of power transistors have increased greatly, making the transistor more desirable in many applications.

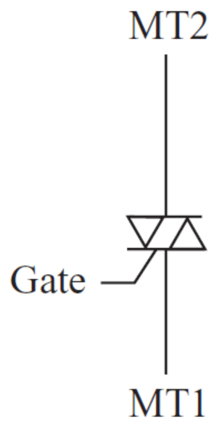


Figure (11-d)
Triac

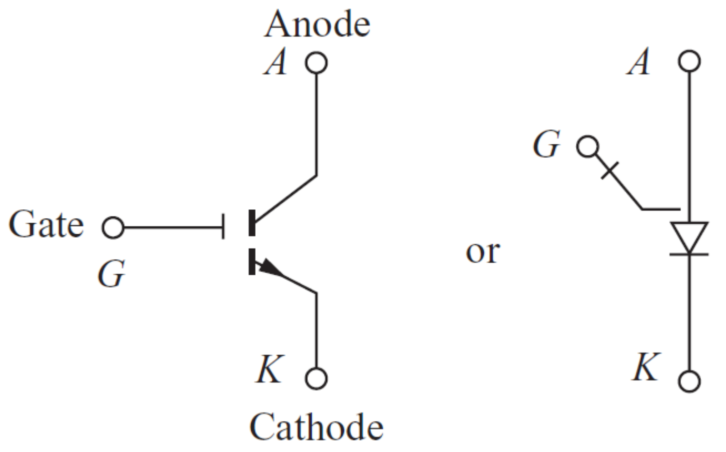


Figure (11-e)
MOS-controlled thyristor (MCT)

Power Bipolar Junction Transistor (BJT) shown in figure (12) is the first semiconductor device to allow full control over its Turn on and Turn off operations. It simplified the design of a large number of Power Electronic circuits that used forced commutated thyristors at that time and also helped realize a number of new circuits. Subsequently, many other devices that can broadly be classified as “Transistors” have been developed. Many of them have superior performance compared to the BJT in some respects. They have, by now, almost completely replaced BJTs. However, it should be emphasized that the BJT was the first semiconductor device to closely approximate an ideal fully controlled Power switch. Other “transistors” have characteristics that are qualitatively similar to those of the BJT (although the physics of operation may differ). Power BJT can be connected in a Darlington configuration to obtain higher collector current as shown in figure (13).

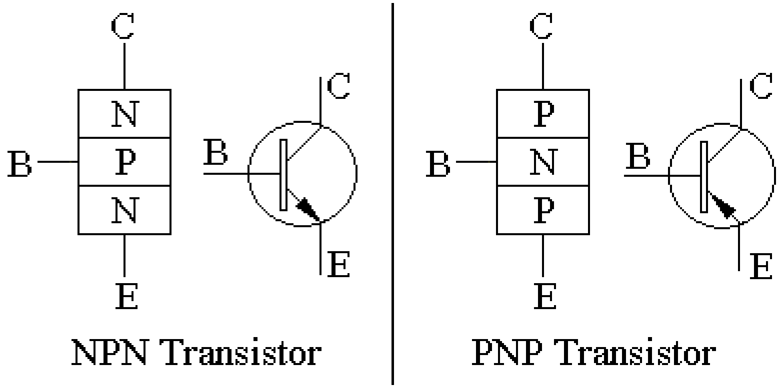


Figure (12) (BJT)

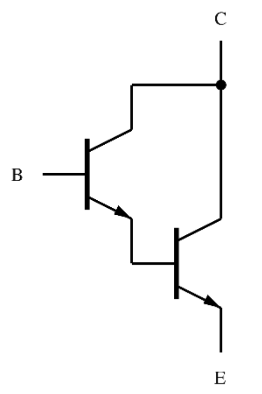


Figure (13)
Darlington configuration

An insulated-gate bipolar transistor (IGBT) shown in figure (14) is a three-terminal power semiconductor device primarily used as an electronic switch which, as it was developed, came to combine high efficiency and fast switching. It switches electric power in many applications: variable-frequency drives (VFDs), electric cars, trains, variable speed refrigerators, lamp ballasts, air-conditioners and even stereo systems with switching amplifiers. The IGBT is a semiconductor device with four alternating layers (P-N-P-N) that are controlled by a metal-oxide-semiconductor (MOS) gate structure without regenerative action. From the input side the IGBT operates as MOSFET. From the output side the IGBT operates somehow like a BJT. The IGBT is faster than a BJT but slower than a MOSFET. The on-state losses are similar to a BJT and lower to a MOSFET. The IGBT is the workhorse in the middle power range in Power Electronics.

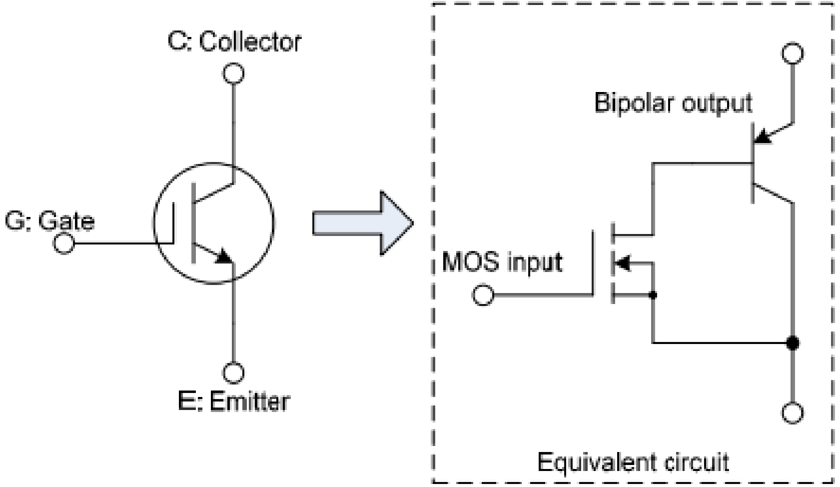


Figure (14) An insulated-gate bipolar transistor (IGBT)

MOSFET: Metal Oxide Semiconductor Field Effect Transistor is shown in figure (15). Today Power MOSFETs are the workhorse in Power Electronics at low power. MOSFETs are normally-off devices and are turned on by applying the gate with sufficient charge. The MOSFET switches on and off fast because it is a majority-carrier device. No charge has to be removed or injected to turn off or turn on the device. At high blocking voltage the on-state loss is high because of the high internal build-in resistance.

The MOSFET can function in two ways:

1- **Depletion Mode**:

When there is no voltage on the gate, the channel shows its maximum conductance. As the voltage on the gate is either positive or negative, the channel conductivity decreases.

2- **Enhancement Mode**:

When there is no voltage on the gate the device does not conduct. More is the voltage on the gate, the better the device can conduct.

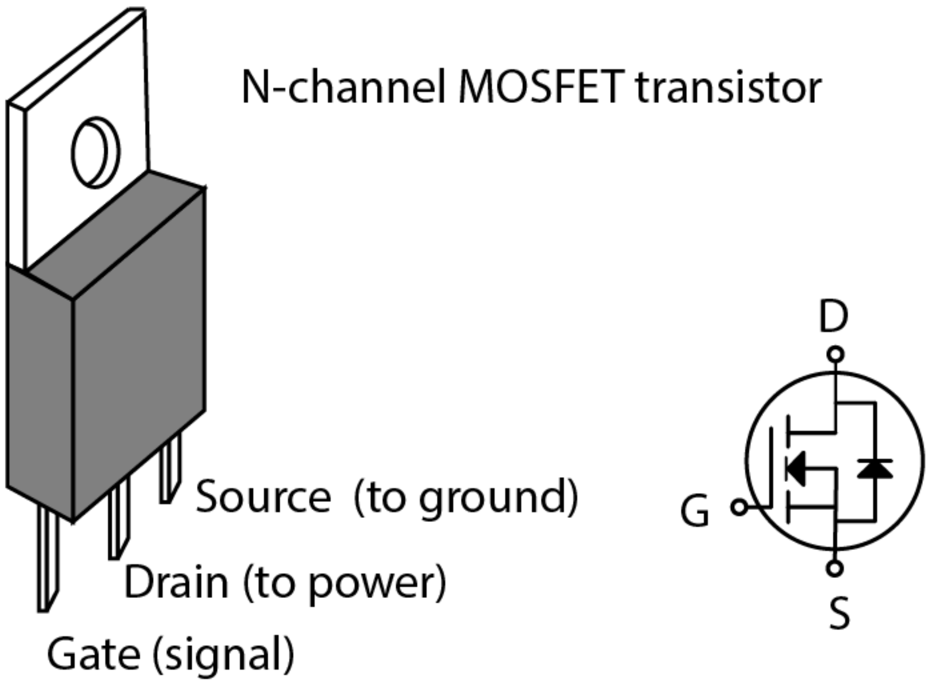


Figure (15) MOSFET: Metal Oxide Semiconductor Field Effect Transistor

device	diode	BJT	IGBT	MOSFET	thyristor	GTO
Max. voltage	50kV	1.8kV	6kV	1.5kV	12kV	6.5kV
Max current	6kA	1kA	1.2kA	1kA	6kA	6kA
Max. frequencies	>1MHz	30kHz	50kHz	>1MHz	10kHz	1kHz
On-sate loss	low	moderate	moderate	high	v. low	low
Switching loss	moderate	moderate	moderate	low	high	high
Drive requirements	none	high	v. low	v. low	low	moderate
Cost	low	low	moderate	moderate	low	high

Comparison of Power Semiconductor Devices

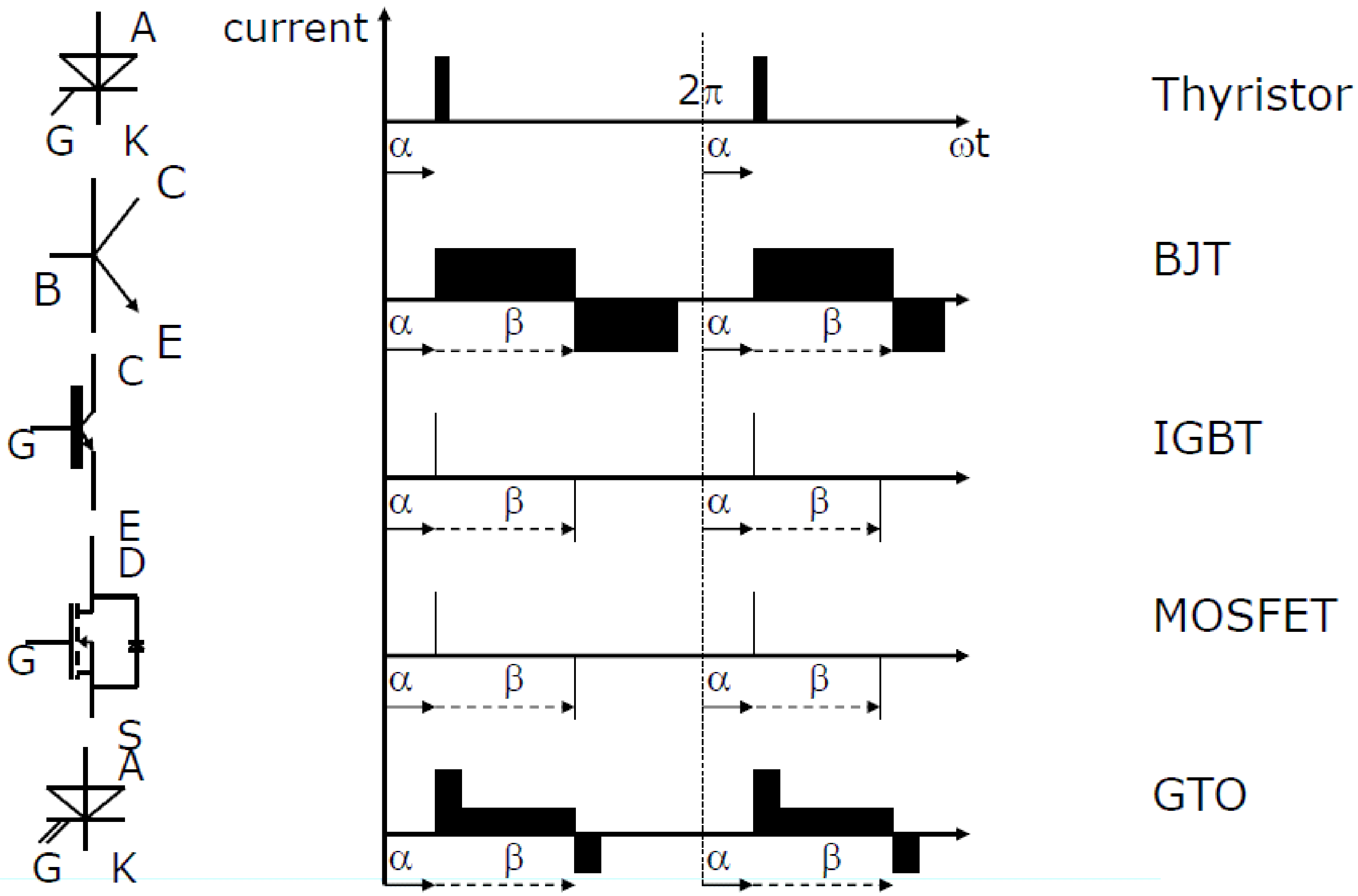


Figure (16) Control signals

Different turn ON methods for SCR

1. Forward voltage triggering
2. Gate triggering
3. dv/dt triggering
4. Light triggering
5. Temperature triggering

Chapter 3
AC to DC converters

A rectifier converts ac to dc. The purpose of a rectifier may be to produce an output that is purely dc, or the purpose may be to produce a voltage or current waveform that has a specified dc component.

Rectifiers

Single-phase rectifiers

- Half-wave Uncontrolled rectifier
- Half-wave controlled rectifier
- Semi-controlled Bridge rectifier
- Full-wave Uncontrolled rectifier
- Full-wave controlled rectifier

Three-phase rectifiers

- Half-wave Uncontrolled rectifier
- Half-wave controlled rectifier
- Semi-controlled Bridge rectifier
- Full-wave Uncontrolled rectifier
- Full-wave controlled rectifier

1- **Single-phase Half-wave Uncontrolled rectifier with Resistive load**

In practice, the half-wave rectifier is used most often in low-power applications because the average current in the supply will not be zero, and nonzero average current may cause problems in transformer performance. While practical applications of this circuit are limited, it is very worthwhile to analyse the halfwave rectifier in detail.

A basic half-wave rectifier with a resistive load is shown in Figure (17-a). The source is ac, and the objective is to create a load voltage that has a nonzero dc component. The diode is a basic electronic switch that allows current in one direction only. For the positive half-cycle of the source in this circuit, the diode is on (forward biased). Considering the diode to be ideal, the voltage across a forward-biased diode is zero and the current is positive. For the negative half-cycle of the source, the diode is reverse-biased, making the current zero. The voltage across the reverse-biased diode is the source voltage, which has a negative value.

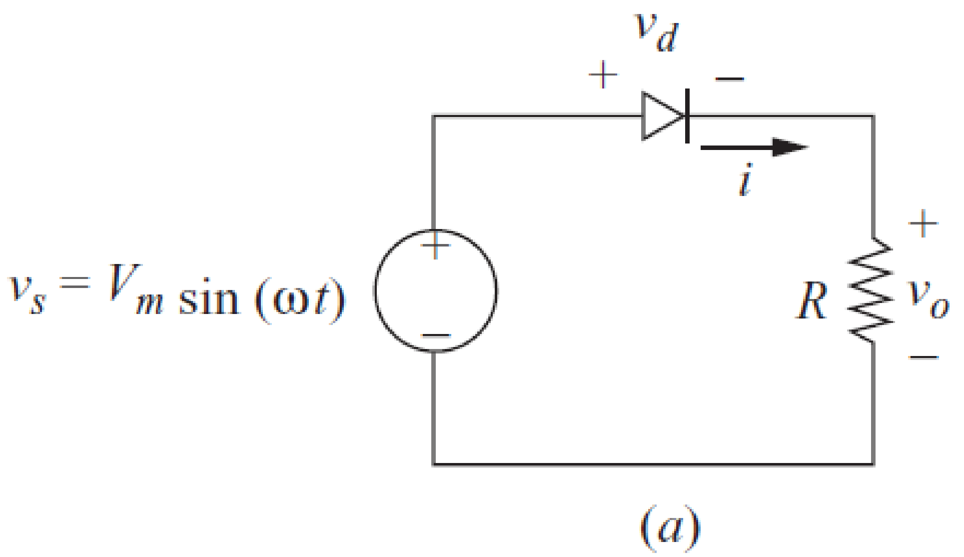


Figure (17-a) half-wave rectifier with a resistive load

The voltage waveforms across the source, load, and diode are shown in Figure (17-b). Note that the units on the horizontal axis are in terms of angle (t). This representation is useful because the values are independent of frequency. The dc component *Vo* of the output voltage is the average value of a half-wave rectified sinusoid.

$$V_o = V_{av} = \frac{1}{2\pi}\int_0^{\pi} V_m \sin wt\, dwt = \frac{V_m}{\pi}$$

The dc component of the current for the purely resistive load is $I_o = \frac{V_o}{R} = \frac{V_m}{\pi R}$

Average power absorbed by the resistor in Figure (17-a) can be computed from $P = {I_{rms}}^2 R = \frac{{V_{rms}}^2}{R}$

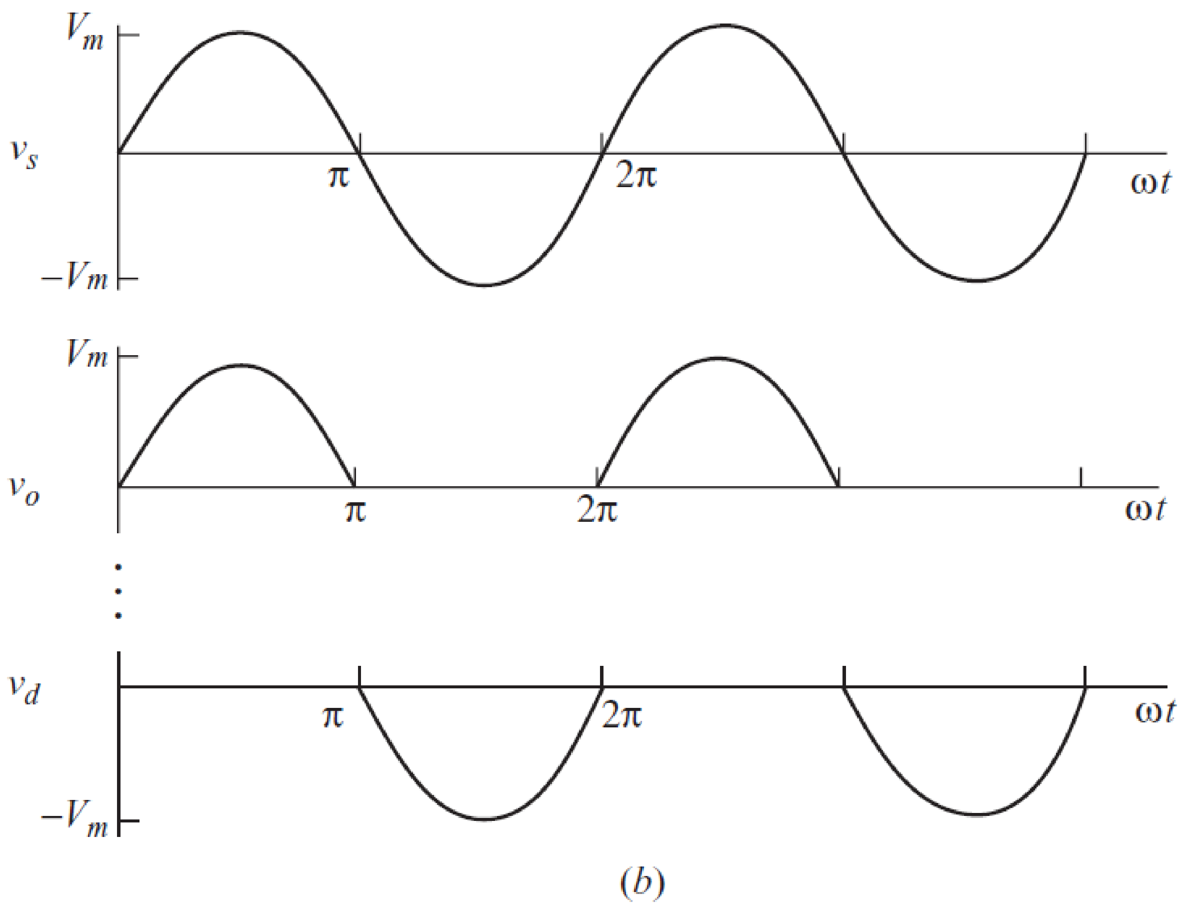


Figure (17-b) The voltage waveforms across the source, load, and diode

When the voltage and current are half-wave rectified sine waves

$$V_{rms} = \sqrt{\frac{1}{2\pi}\int_0^{\pi}(V_m \sin wt)^2 dwt} = \frac{V_m}{2}$$

$$I_{rms} = \frac{V_m}{2R}$$

In the preceding discussion, the diode was assumed to be ideal. For a real diode, the diode voltage drop will cause the load voltage and current to be reduced, but not appreciably if Vm is large. For circuits that have voltages much larger than the typical diode drop, the improved diode model may have only second-order effects on the load voltage and current computations.

Example 3-1

2- **Single-phase Half-wave controlled rectifier**

A- with Resistive load

Away to control the output of a half-wave rectifier is to use an SCR instead of a diode. Figure (18-a) shows a basic controlled half-wave rectifier with a resistive load. Two conditions must be met before the SCR can conduct:

1. The SCR must be forward-biased (V SCR > 0).
2. A current must be applied to the gate of the SCR.

Unlike the diode, the SCR will not begin to conduct as soon as the source becomes positive. Conduction is delayed until a gate current is applied, which is the basis for using the SCR as a means of control. Once the SCR is conducting, the gate current can be removed and the SCR remains on until the current goes to zero.

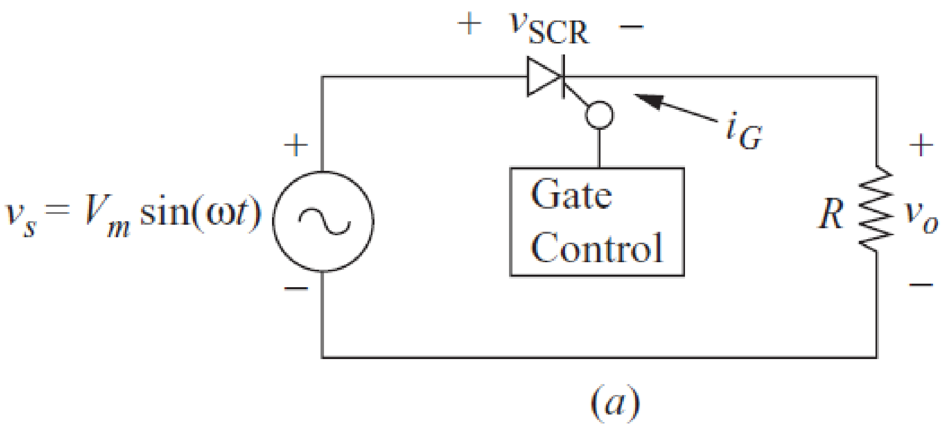


Figure (18-a) basic controlled half-wave rectifier with a resistive load

Figure (18-b) shows the voltage waveforms for a controlled half-wave rectifier with a resistive load. Agate signal is applied to the SCR at *t*, where is the delay angle.

The average (dc) voltage across the load resistor in Figure (13-a) is

$$V_o = \frac{1}{2\pi}\int_{\alpha}^{\pi} V_m \sin wt\, dwt = \frac{V_m}{2\pi}(1+\cos\alpha)$$

The power absorbed by the resistor is $\frac{V_{rms}{}^2}{R}$

where the rms voltage across the resistor is computed from

$$V_{rms} = \sqrt{\frac{1}{2\pi}\int_{0}^{2\pi} V_o{}^2\, wt\, dwt}$$

$$= \sqrt{\frac{1}{2\pi}\int_{\alpha}^{\pi} (V_m \sin wt)^2\, wt\, dwt}$$

$$= \frac{V_m}{2}\sqrt{1-\frac{\alpha}{\pi}+\frac{\sin 2\alpha}{2\pi}}$$

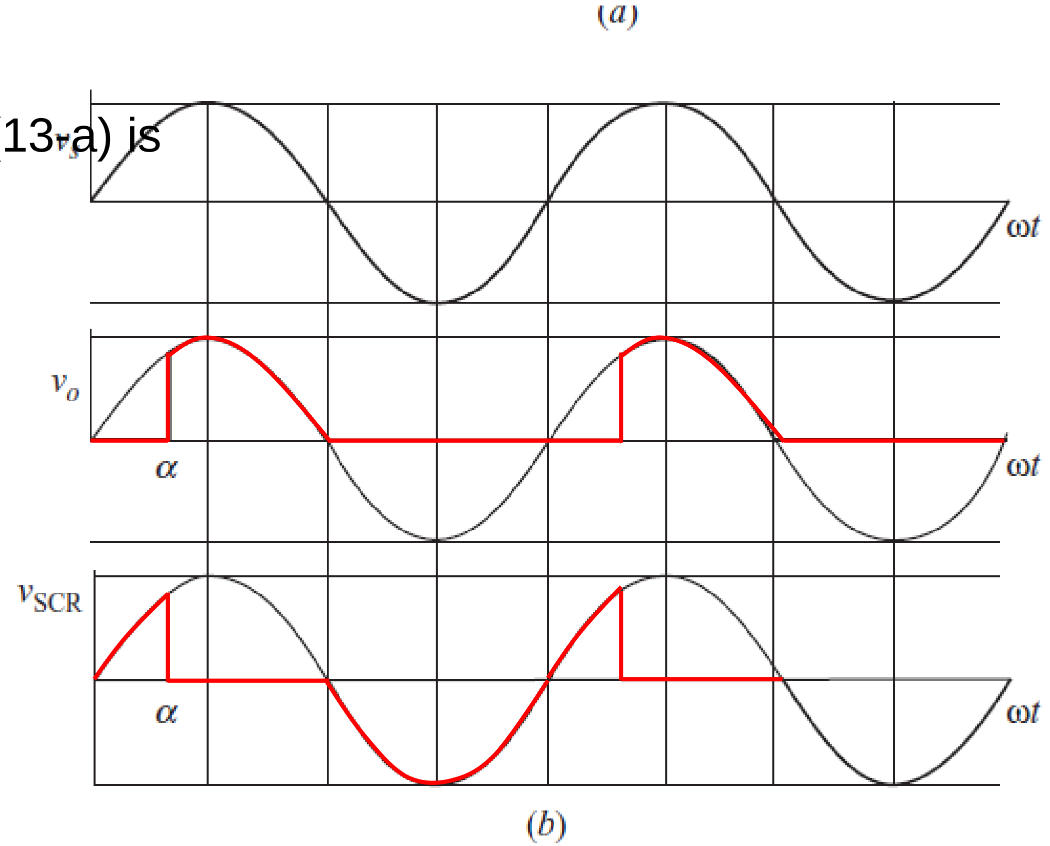


Figure (18-b) voltage waveforms for a controlled half-wave rectifier with a resistive load

Example 3.2

B- with RL load

A controlled half-wave rectifier with an *RL* load is shown in Figure (19-a). The analysis of this circuit is similar to that of the uncontrolled rectifier. The current is the sum of the forced and natural responses.

$$i(wt) = if(wt) + in(wt) = \frac{V_m}{Z}\, sin\,(wt - \theta) + Ae^{-wt/w\tau}$$

The constant A is determined from the initial condition $i(\alpha) = 0$

$$i(\alpha) = 0 = \frac{V_m}{Z}\sin(\alpha - \theta) + Ae^{-\alpha/w\tau}$$

$$A = \left[-\frac{V_m}{Z}\sin(\alpha - \theta)\right] * e^{\alpha/w\tau}$$

Substituting for A and simplifying,

$$i(wt) = \begin{cases} \frac{V_m}{Z}\left[\sin(wt - \theta) - \sin(\alpha - \theta)e^{(\alpha - wt)/w\tau}\right] & For\ \alpha \le wt \le \beta \\ 0 & Otherwise \end{cases}$$

$$V_o rms = \frac{Vm}{2\sqrt{\pi}}\sqrt{(\beta - \alpha) - \frac{1}{2}(\sin 2\beta - \sin 2\alpha)}$$

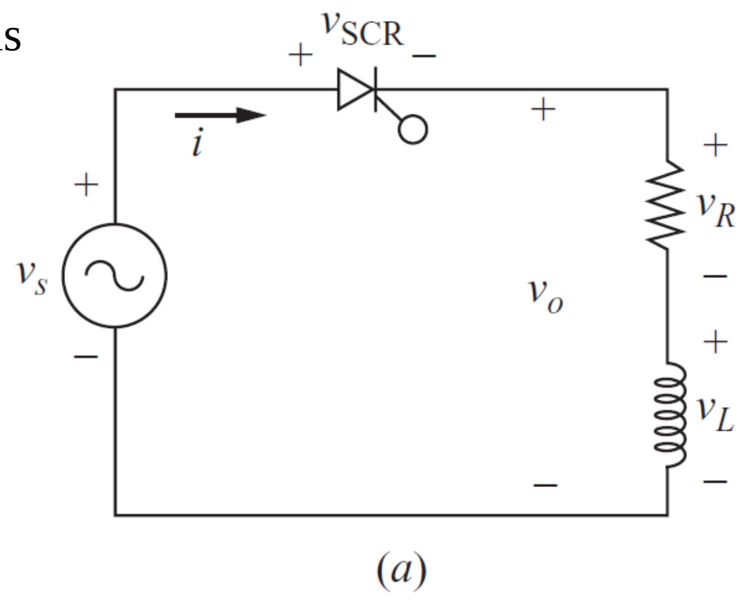


Figure (19-a) A controlled half-wave rectifier with an *RL*

The extinction angle is defined as the angle at which the current returns to zero, as in the case of the uncontrolled rectifier. When $wt = \beta$ which must be solved numerically for β. The angle is called the conduction angle γ.

$$i(\beta) = 0 = \frac{V_m}{Z}\left[\sin(\beta - \theta) - \sin(\alpha - \theta)e^{(\alpha-\beta)/w\tau}\right]$$

Figure (19-b) shows the voltage waveforms.

The average (dc) output voltage is

$$V_o av = \frac{1}{2\pi}\int_{\alpha}^{\beta} V_m \sin wt\, dwt = \frac{V_m}{2\pi}(\cos\alpha - \cos\beta)$$

The average current is computed from

$$I_o = \frac{1}{2\pi}\int_{\alpha}^{\beta} i(wt)dwt$$

Power absorbed by the load is $I_{rms}{}^{2}R$, where the rms current is computed from

$$I_{rms} = \sqrt{\frac{1}{2\pi}\int_{\alpha}^{\beta} i^2(wt)dwt}$$

Example 3.3

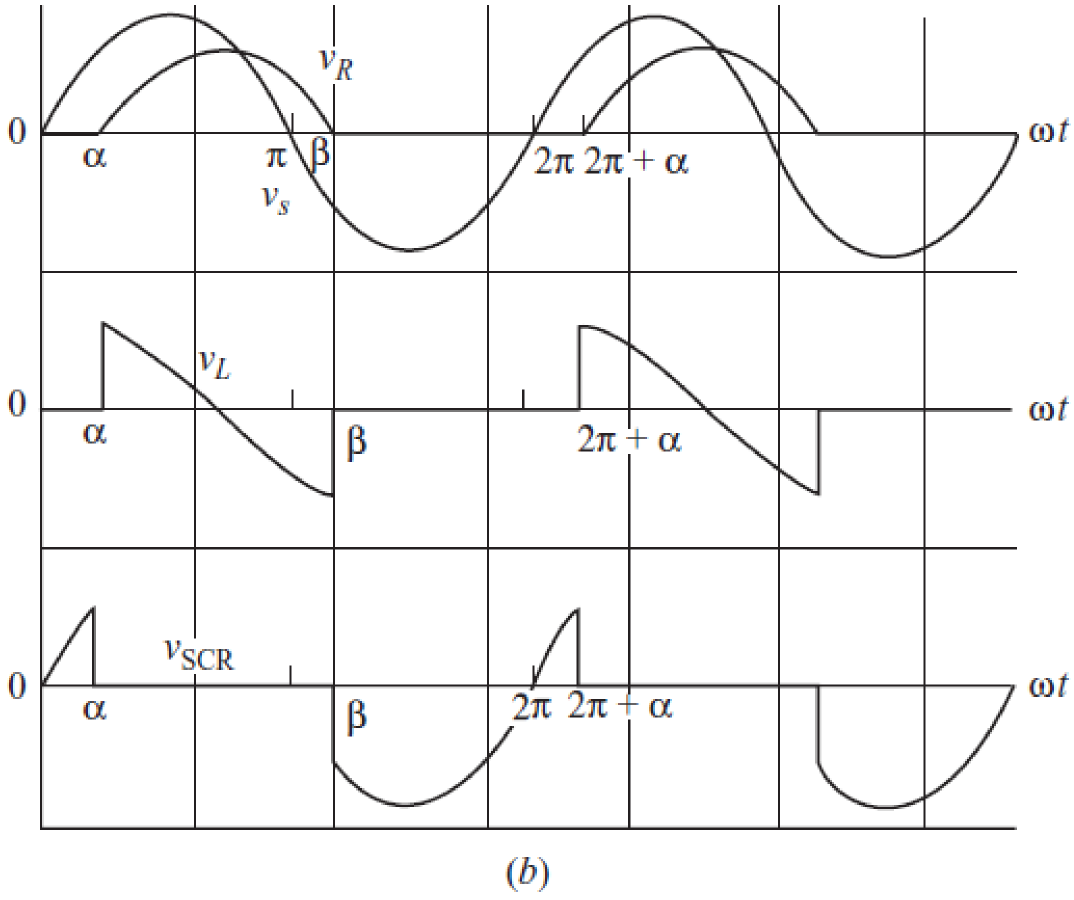


Figure (19-b) voltage waveforms of controlled half-wave rectifier with an *RL*

3- **Single-phase Semi-controlled Bridge rectifier**
A- with Resistive load

The Single-phase Semi-controlled Bridge rectifier circuit with resistive load is shown in figure (20-a). Single phase half controlled converters are obtained from fully controlled converters by replacing two thyristors by two diodes. There are four operating modes of this converter when current flows through the load. Of course it is always possible that none of the four devices conduct. The load current during such periods will be zero. Half controlled converters are most favoured in applications requiring unidirectional output voltage and current. Figure (20-b) shows the output voltage waveform in case of a resistive load.

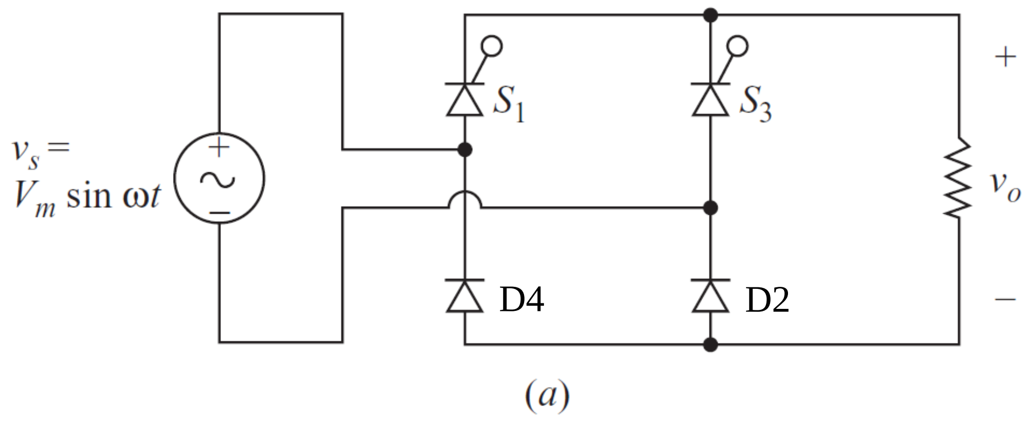


Figure (20-a). Single-phase Semi-controlled Bridge rectifier circuit with resistive load

During positive half cycle thyristor S1 and diode D2 conduct and in negative half cycle thyristor S3 and diode D4 conduct. As the conduction is through diode, the output voltage wave during R load will not extend in the negative direction as shown in figure (19-b).

The average output voltage value will be

$$V_o av = \frac{V_m}{\pi}(1 + \cos\alpha)$$

$$V_o rms = Vm\sqrt{\frac{1}{2} - \frac{\alpha}{2\pi} + \frac{\sin 2\alpha}{4\pi}}$$

where Vm is the maximum value of the ac wave, and α is the firing angle

In a half controlled converter the output voltage does not become negative and hence the converter cannot operate in the inverter mode.

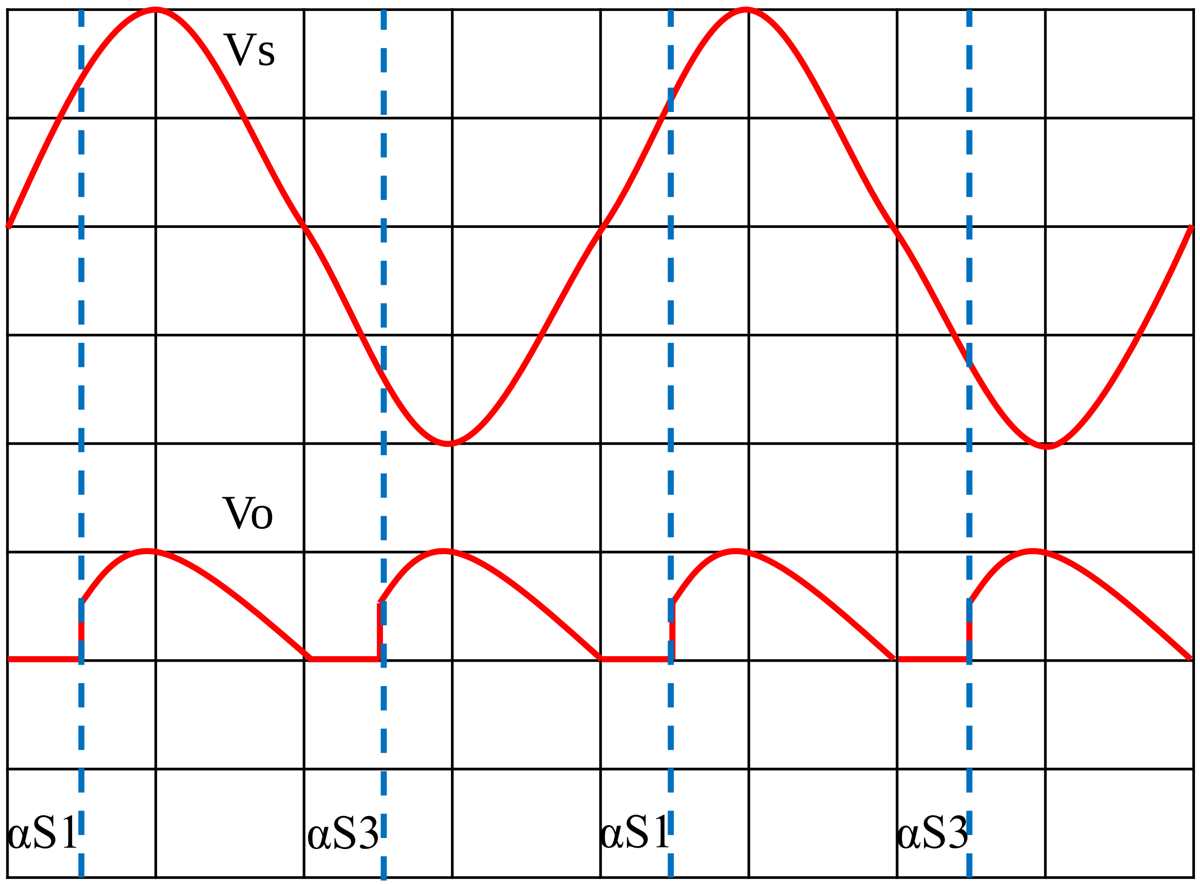


Figure (19-b) voltage waveforms of semi-controlled rectifier with resistive load

B- with RL load

In the case of RL load, the output current wave will extend in the negative portion but still positive, as shown in figure (19-c).

Average output current is then

$$I_o av = \frac{V_o}{R} = \frac{V_m}{\pi R}(1 + \cos\alpha)$$

$$V_o rms = Vm\sqrt{\frac{1}{2} - \frac{\alpha}{2\pi} + \frac{\sin 2\alpha}{4\pi}}$$

$$I_o rms = \frac{V_{o\ rms}}{Z}$$

$$were\ z = \sqrt{R^2 + (wL)^2}$$

The power delivered to the load is a function of the input voltage, the delay angle, and the load components; $P = {I_{rms}}^2$ R is used to determine the power in a resistive load

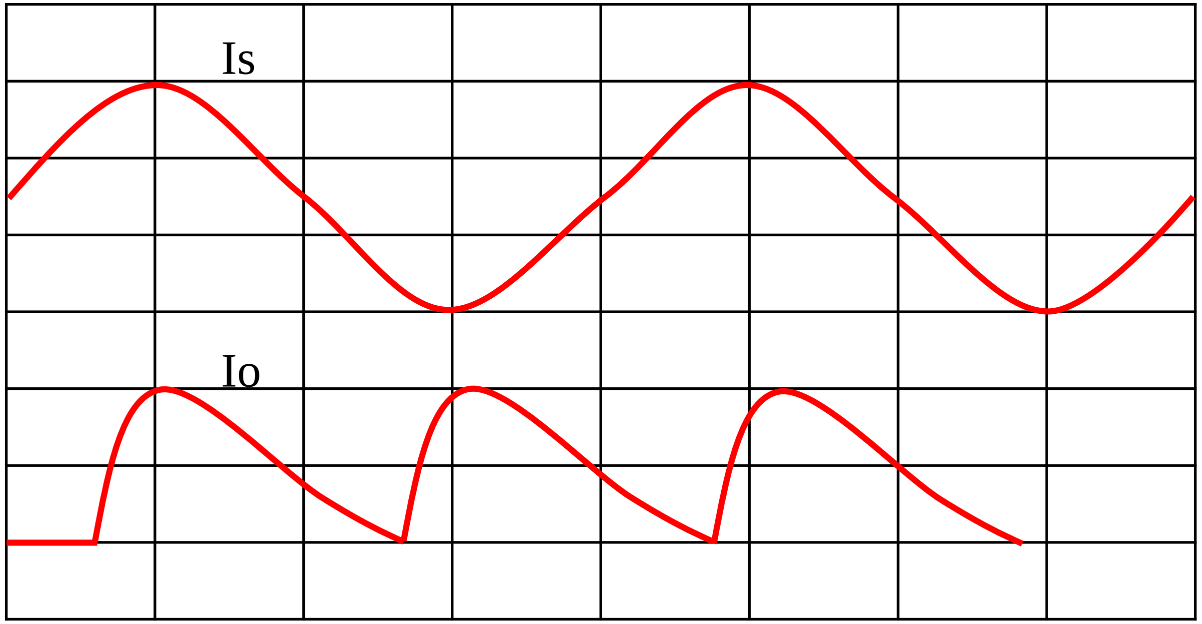


Figure (19-c) Single-phase Semi-controlled Bridge rectifier with RL load

4- Single-phase Full-wave controlled rectifier

A- with Resistive load.

A versatile method of controlling the output of a full-wave rectifier is to substitute controlled switches such as thyristors (SCRs) for the diodes. Output is controlled by adjusting the delay angle of each SCR, resulting in an output voltage that is adjustable over a limited range. Controlled full-wave rectifiers are shown in Figure (20-a). For the bridge rectifier, SCRs S1 and S2 will become forward-biased when the source becomes positive but will not conduct until gate signals are applied. Similarly, S3 and S4 will become forward-biased when the source becomes negative but will not conduct until they receive gate signals. The delay angle is the angle interval between the forward biasing of the SCR and the gate signal application. If the delay angle is zero, the rectifiers behave exactly as uncontrolled rectifiers with diodes.

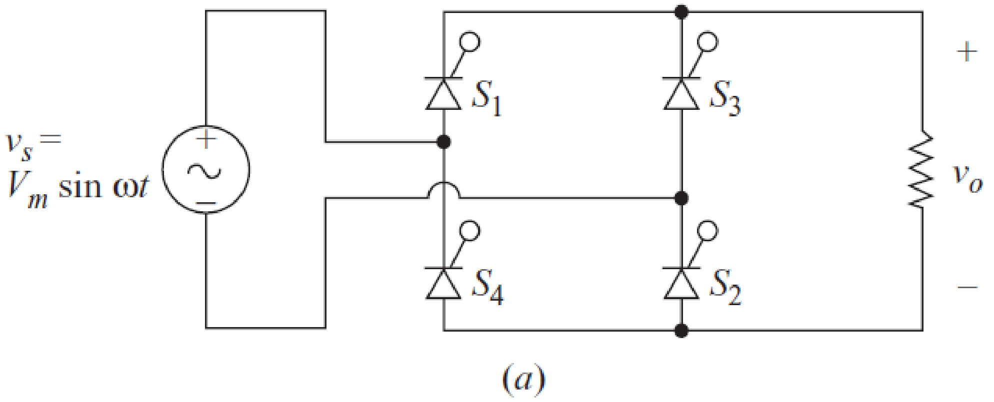


Figure (20-a) Controlled full-wave rectifiers with a resistive load

The output voltage waveform for a controlled full-wave rectifier with a resistive load is shown in Figure (20-b). The average component of this waveform is determined from

$$V_oav = \frac{1}{\pi}\int_{\alpha}^{\pi} V_m \sin wt \; dwt = \frac{V_m}{\pi}(1 + \cos\alpha)$$

Average output current is then

$$I_o = \frac{V_o}{R} = \frac{V_m}{\pi R}(1 + \cos\alpha)$$

The power delivered to the load is a function of the input voltage, the delay angle, and the load components; $P = {I_{rms}}^2$ R is used to determine the power in a resistive load, where

$$I_{rms} = \sqrt{\frac{1}{\pi}\int_{\alpha}^{\pi}\left(\frac{V_m}{R}\sin wt\right)^2 dwt} = \frac{V_m}{R}\sqrt{\frac{1}{2} - \frac{\alpha}{2\pi} + \frac{\sin 2\alpha}{4\pi}}$$

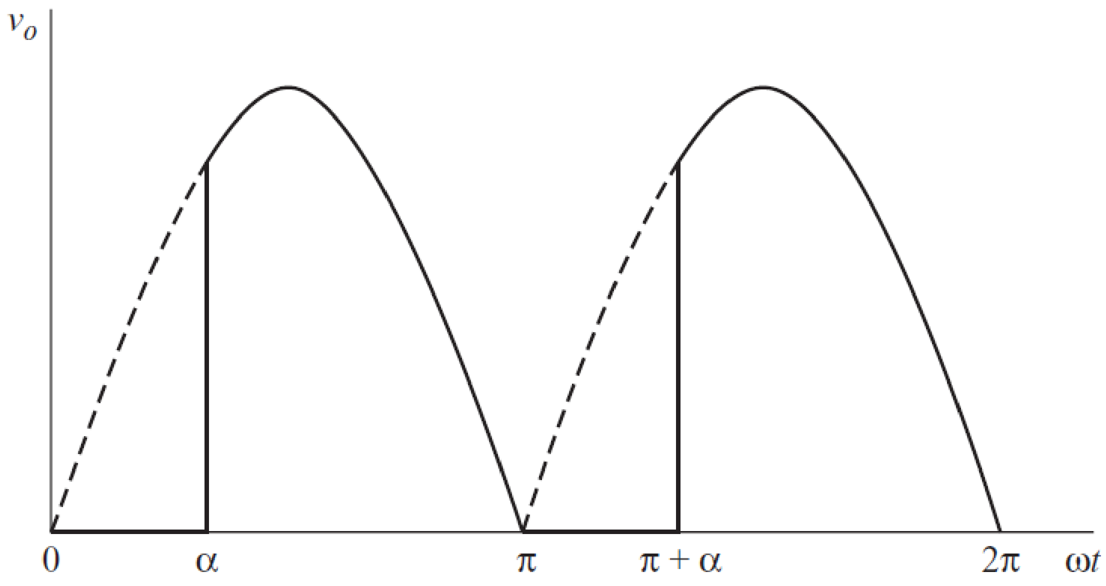


Figure (20-b) output voltage waveform for a controlled full-wave rectifier with a resistive load

The rms current in the source is the same as the rms current in the load. Example 3.4

B- with RL load. Discontinuous Current

Load current for a controlled full-wave rectifier with an RL load figure (21-a) be either continuous or discontinuous, and a separate analysis is required for each. Starting the analysis at $wt = 0$ with zero load current, SCRs S1 and S2 in the bridge rectifier will be forward-biased and S3 and S4 will be reverse-biased as the source voltage becomes positive. Gate signals are applied to S1 and S2 at $wt = \alpha$ turning S1 and S2 on. With S1 and S2 on, the load voltage is equal to the source voltage. For this condition, the circuit is identical to that of the controlled halfwave rectifier having a current function

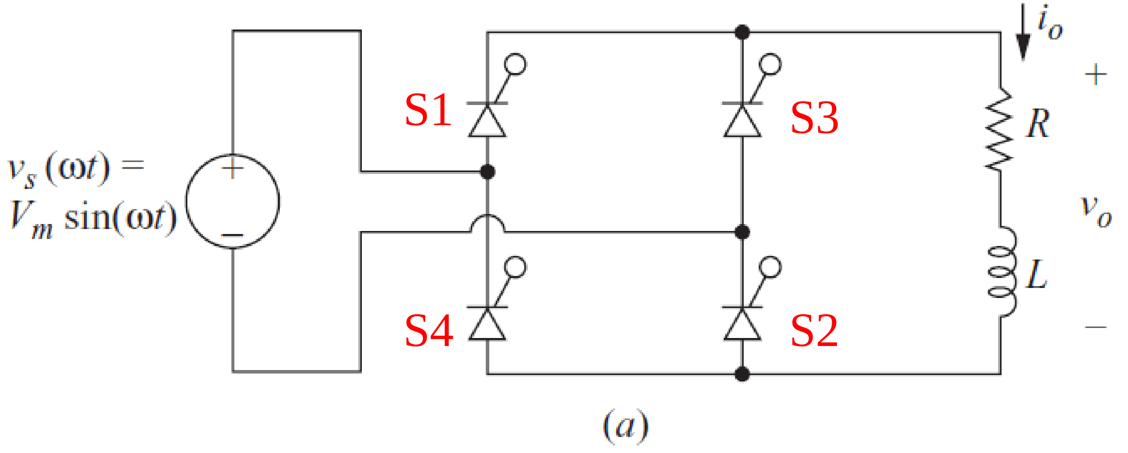


Figure (21-a) controlled full-wave rectifier with an RL load

$$i_o(wt) = \frac{V_m}{Z}\left[\sin(wt-\theta) - \sin(\alpha-\theta)e^{-(wt-\alpha)/w\tau}\right]$$

for $\alpha \le wt \le \beta$

Where $Z = \sqrt{R^2 + (wl)^2}$ $\qquad \theta = tan^{-1}\left(\frac{wl}{R}\right)$ $\qquad \tau = \frac{L}{R}$

The above current function becomes zero at $wt = \beta$. If $\beta < (\pi + \alpha)$, the current remains at zero until $wt = (\pi + \alpha)$ when gate signals are applied to $S3$ and $S4$ which are then forward-biased and begin to conduct. This mode of operation is called *discontinuous current*, which is illustrated in figure (21-b).

If $\beta < (\pi + \alpha) \Longrightarrow discontinuous\ current$

Analysis of the controlled full-wave rectifier operating in the discontinuous current mode is identical to that of the controlled half-wave rectifier except that the period for the output current is π rather than 2π rad.

$$V_o rms = \frac{Vm}{\sqrt{2\pi}} \sqrt{(\beta - \alpha) - \frac{1}{2}(\sin 2\beta - \sin 2\alpha)}$$

$$V_o av = \frac{V_m}{\pi}(\cos\alpha - \cos\beta)$$

Example 3.5

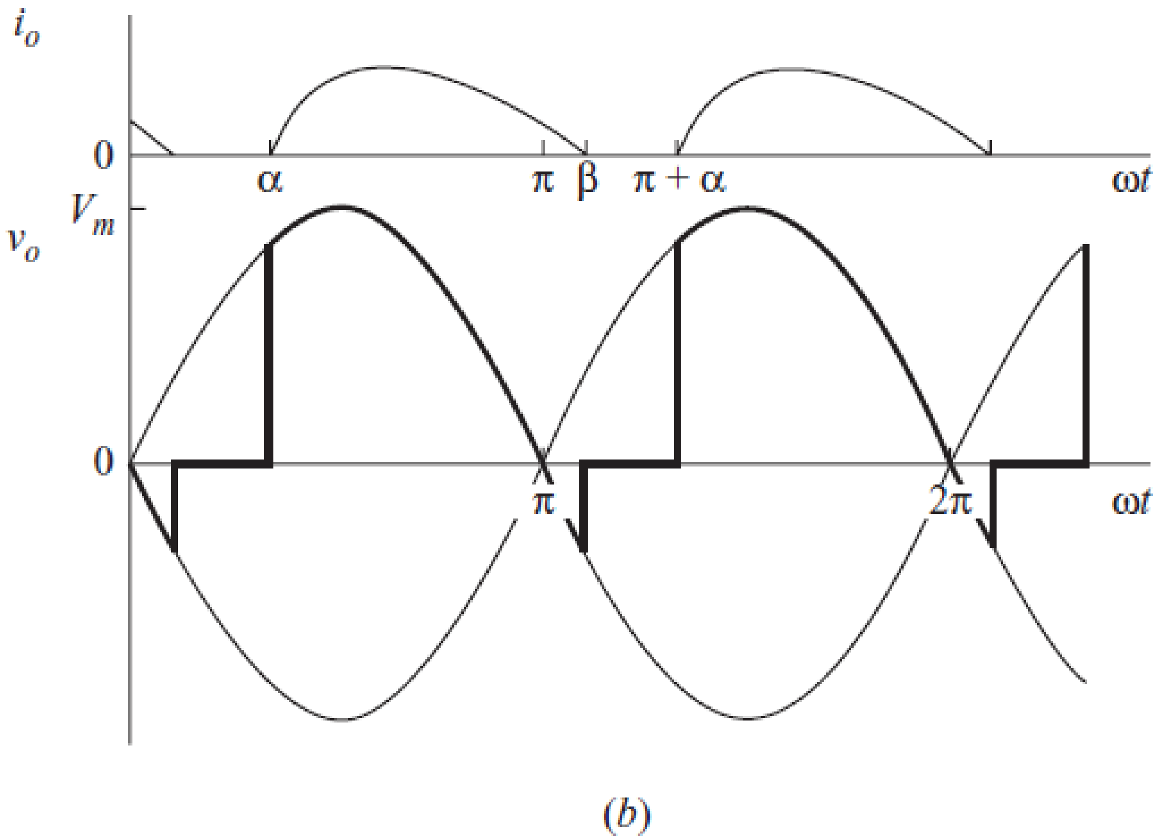


Figure (21-b) current and voltage waveforms discontinuous current mode

C- with RL load. Continuous Current

If the load current is still positive at $wt = (\pi + \alpha)$ when gate signals are applied to S3 and S4 in the above analysis, S3 and S4 are turned on and S1 and S2 are forced off. Since the initial condition for current in the second half-cycle is not zero, the current function does not repeat. For an RL load with continuous current, the steady-state current and voltage waveforms are generally as shown in Figure (21-c). The boundary between continuous and discontinuous current occurs when $\beta = (\pi + \alpha)$ The current at $wt = (\pi + \alpha)$ must be greater than zero for continuous-current operation.

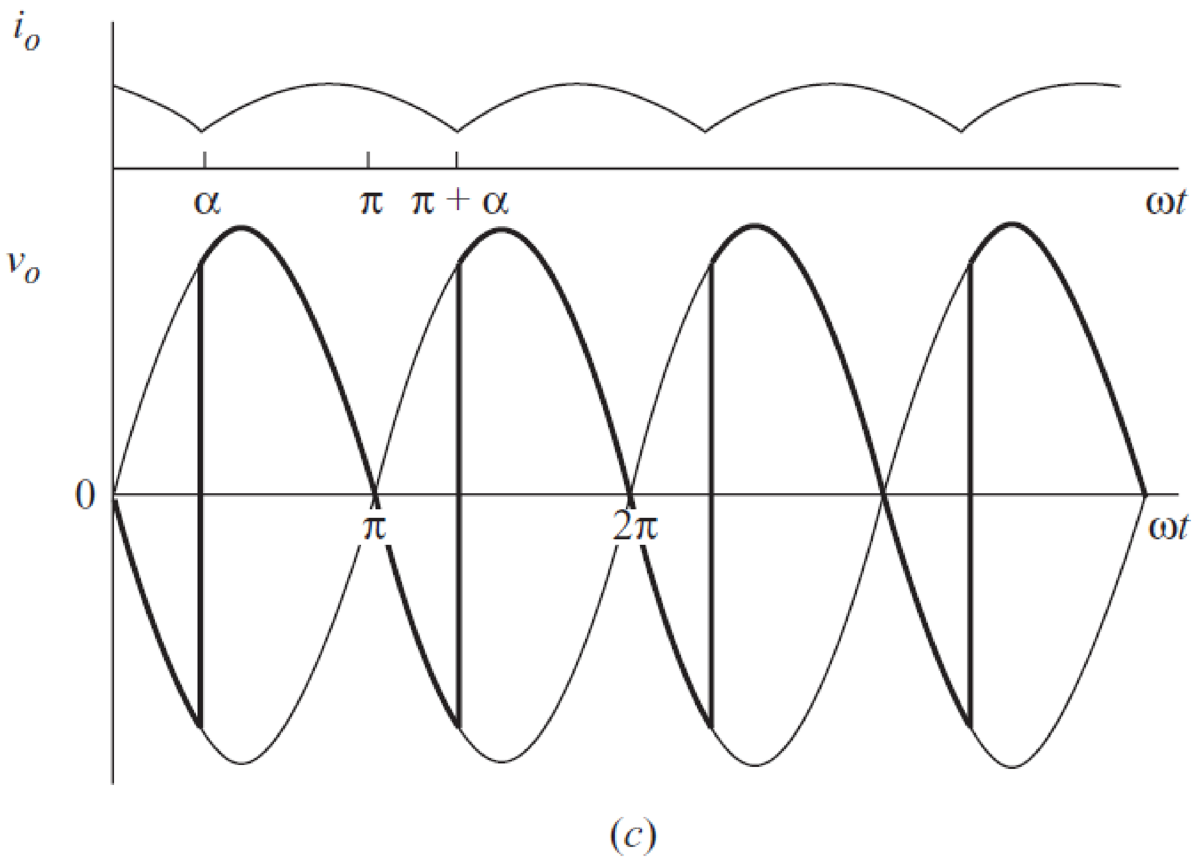


Figure (21-c) RL load with continuous current, the steady-state current and voltage waveforms

$i(\pi+\alpha) \geq 0$

$\sin(\pi+\alpha-\theta) - \sin(\pi+\alpha-\theta)e^{-(\pi+\alpha-\alpha)/w\tau} \geq 0$

using

$\sin(\pi+\alpha-\theta) = \sin(\theta-\alpha)$

$\sin(\theta-\alpha)\left(1-e^{-\left(\frac{\pi}{w\tau}\right)}\right) \geq 0$

Solving for α $\alpha \leq \theta$

using

$\theta = tan^{-1}\left(\frac{wl}{R}\right)$ $\alpha \leq tan^{-1}\left(\frac{wl}{R}\right)$ for continuous current

Either $\beta < (\pi+\alpha)$ or $\alpha \leq tan^{-1}\left(\frac{wl}{R}\right)$ can be used to check whether the load current is continuous or discontinuous.

A method for determining the output voltage and current for the continuous current case is to use the Fourier series. The Fourier series for the voltage waveform for continuous-current case shown in Figure (21-c) is expressed in general form as:

$$V_o(wt) = V_o + \sum_{n=1}^{\infty} V_n \cos(n\, w_o\, t + \theta_n)$$

The dc (average) value is

$$V_o av = \frac{1}{\pi}\int_{\alpha}^{\alpha+\pi} V_m \sin wt\; dwt = \frac{2V_m}{\pi}\cos\alpha$$

The amplitudes of the ac terms are calculated from $V_n = \sqrt{{a_n}^2 + {b_n}^2}$

$$a_n = \frac{2V_m}{\pi}\left[\frac{\cos(n+1)\alpha}{n+1} - \frac{\cos(n-1)\alpha}{n-1}\right]$$

$$b_n = \frac{2V_m}{\pi}\left[\frac{\sin\,(n+1)\alpha}{n+1} - \frac{\sin\,(n-1)\alpha}{n-1}\right]$$

$Where\; n = \; 2, 4, 6, \ldots$

Figure (22) shows the relationship between normalized harmonic content of the output voltage and delay angle. The Fourier series for current is determined by superposition. The current amplitude at each frequency is determined from

$$I_o av = \frac{V_o av}{R}$$

$$I_n = \frac{V_n}{Z_n} = \frac{V_n}{|R + j\, n\, w_o l|}$$

The rms current is determined by combining the rms currents at each frequency.

$$I_{rms} = \sqrt{I_o av^2 + \sum_{n=2,4,6,\ldots}^{\infty} \left(\frac{I_n}{\sqrt{2}}\right)^2}$$

Homework 3.1/3.6

As the harmonic number increases, the impedance for the inductance increases. Therefore, it may be necessary to solve for only a few terms of the series to be able to calculate the rms current. If the inductor is large, the ac terms will become small, and the current is essentially dc.

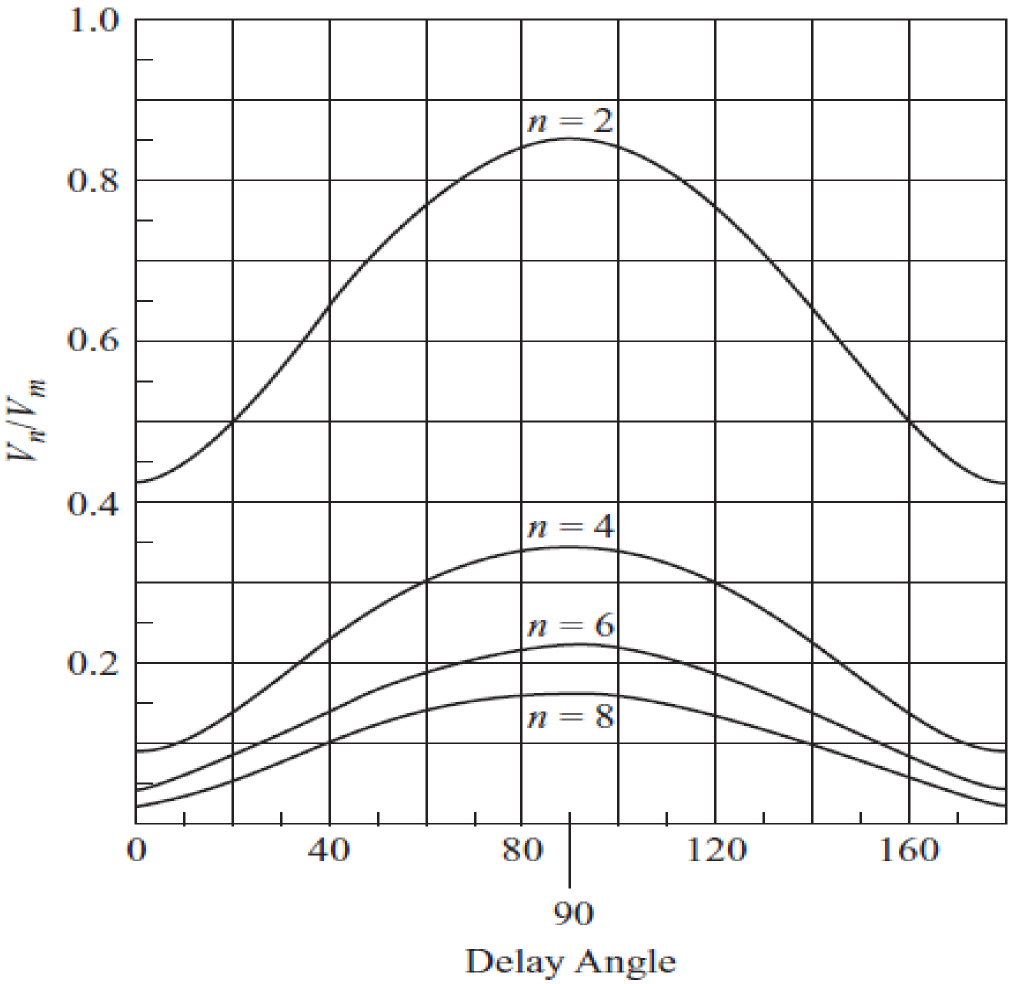


Figure (22) the relationship between normalized harmonic content of the output voltage and delay angle

Homework sheet no.1

5- **Three-phase Half-wave Controlled rectifier**

Single phase half controlled bridge converters & fully controlled bridge converters are used extensively in industrial applications up to about 15kW of output power. The single phase controlled rectifiers provide a maximum dc output of $V_{dc\,(\max)} = \frac{2V_m}{\pi}$. The output ripple frequency is equal to the twice the ac supply frequency. The single phase full wave controlled rectifiers provide two output pulses during every input supply cycle and hence are referred to as two pulse converters. Three phase converters are 3-phase controlled rectifiers which are used to convert ac input power supply into dc output power across the load.

Features of 3-phase controlled rectifiers are

- Operate from 3 phase ac supply voltage
- They provide higher dc output voltage and higher dc output power
- Higher output voltage ripple frequency
- Filtering requirements are simplified for smoothing out load voltage and load current
- Three phase controlled rectifiers are extensively used in high power variable speed industrial dc drives

Three single phase half-wave converters are connected together to form a three phase half-wave converter as shown in figure (23-a).

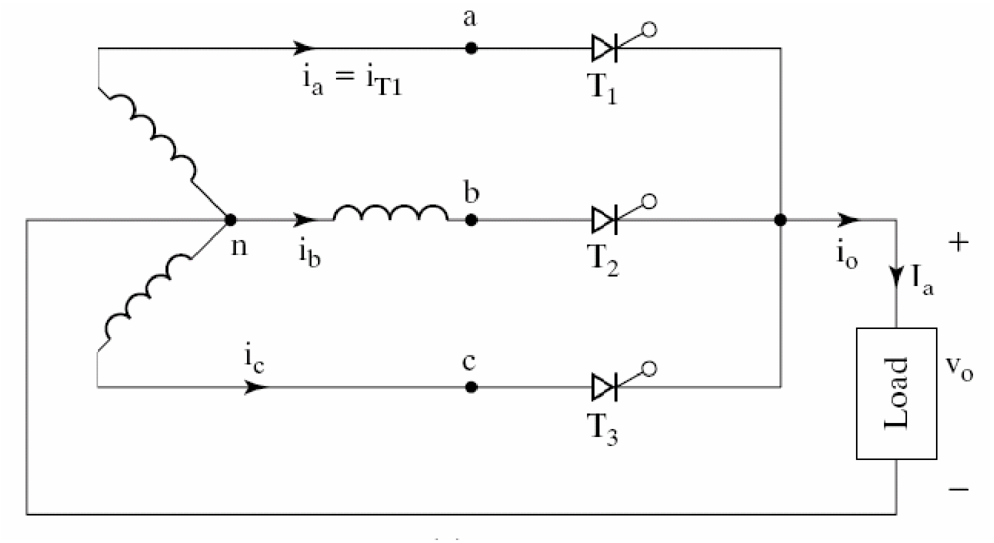


Figure (23-a) three phase half-wave controlled rectifier

5.1- Thee phase supply voltage equations

We define three line neutral voltages (3 phase voltages) as follows:

$$If\ \ V_m = Max.Phase\,Voltage$$

$$V_{RN} = V_{an} = V_m \sin wt$$

$$V_{YN} = V_{bn} = V_m \sin\left(wt - \frac{2\pi}{3}\right)$$

$$V_{BN} = V_{cn} = V_m \sin\left(wt + \frac{2\pi}{3}\right)$$

As shown in figure (23-b)

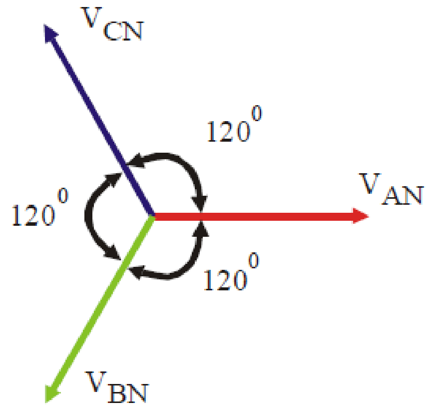


Figure (23-b) Vector diagram of 3-phase supply voltages

The 3-phase half wave converter combines three single phase half wave controlled rectifiers in one single circuit feeding a common load. The thyristor 1 T in series with one of the supply phase windings '$a - n$ ' acts as one half wave controlled rectifier. The second thyristor 2 T in series with the supply phase winding '$b - n$ ' acts as the second half wave controlled rectifier. The third thyristor 3 T in series with the supply phase winding '$c - n$' acts as the third half wave controlled rectifier.

The 3-phase input supply is applied through the star connected supply transformer as shown in figure (23-a). The common neutral point of the supply is connected to one end of the load while the other end of the load connected to the common cathode point.

When thyristor T1 is triggered at $wt = (30 + \alpha)$, the phase voltage V_{an} appears across the load when T1 conducts. The load current flows through the supply phase winding 'a − n ' and through thyristor T1 as long as T1 conducts.

When thyristor T2 is triggered at $wt = (150 + \alpha)$, T1 becomes reverse biased and turns-off. The load current flows through the thyristor T2 and through the supply phase winding 'b − n ' . When T2 conducts the phase voltage V_{bn} appears across the load until the thyristor T3 is triggered .

When the thyristor T3 is triggered at $wt = (270 + \alpha)$, *T2* is reversed biased and hence *T2* turns-off. The phase voltage V_{cn} appears across the load when *T3* conducts.

The 3-phase half wave converter is not normally used in practical converter systems because of the disadvantage that the supply current waveforms contain dc components (i.e., the supply current waveforms have an average or dc value).

5.2- The average and rms output voltages of a 3-phase half wave converter for continuous load current (RL load)

$$V_{dc}=\frac{3}{2\pi}\left[\int_{\frac{\pi}{6}+\alpha}^{\frac{5\pi}{6}+\alpha} Vo\ dwt\right] \qquad where \quad Vo=V_{an}=V_m \sin \mathrm{wt} \qquad for\ wt=(30+\alpha)\ to\ (150+\alpha)$$

$$V_{dc}=\frac{3}{2\pi}\left[\int_{\frac{\pi}{6}+\alpha}^{\frac{5\pi}{6}+\alpha} V_m \sin \mathrm{wt}\ dwt\right] \Longrightarrow V_{dc}=\frac{3\sqrt{3}V_m}{2\pi}\cos\alpha \Longrightarrow V_{dc}=\frac{3V_{Lm}}{2\pi}\cos\alpha$$

Where $V_{Lm}=\sqrt{3}V_m = Max.\,line\ to\ line\ supply\ voltage\ for\ a\ 3-phase\ star\ connected\ transformer.$
And V_m is the peak phase voltage.

$$Vo(RMS)=\sqrt{\frac{3}{2\pi}\int_{\frac{\pi}{6}+\alpha}^{\frac{5\pi}{6}+\alpha} {V_m}^2 sin^2 wt\ dwt} \qquad Vo(RMS)=\sqrt{3}V_m\sqrt{\frac{1}{6}+\frac{\sqrt{3}}{8\pi}\cos 2\alpha}$$

5.3- The average and rms output voltages of a 3-phase half wave converter for discontinuous load current (Resistive load only)

For a purely resistive load where the load inductance 'L = 0' and the trigger angle $\alpha > \left(\frac{\pi}{6}\right)$, the load current appears as discontinuous load current and each thyristor is naturally commutated when the polarity of the corresponding phase supply voltage reverses. The frequency of output ripple frequency for a 3-phase half wave converter is 3 fs, where fs is the input supply frequency.

a) **For firing angle** (0°< α < 30 °)

$$V_{dc} = \frac{3\sqrt{3}V_m}{2\pi}\cos\alpha \qquad V_{dc} = \frac{3V_{Lm}}{2\pi}\cos\alpha$$

$$Vo(RMS) = \sqrt{3}V_m\sqrt{\frac{1}{6} + \frac{\sqrt{3}}{8\pi}\cos 2\alpha} \qquad Io(RMS) = \frac{Vo(RMS)}{R}$$

b) **For firing angle** (30°< α < 90 °)

$$V_{dc} = \frac{3V_m}{2\pi}[1 + \cos(\alpha + 30)] \qquad V_{dc} = \frac{\sqrt{3}V_{Lm}}{2\pi}[1 + \cos(\alpha + 30)]$$

$$Vo(RMS) = \frac{\sqrt{3}V_m}{2\sqrt{\pi}}\sqrt{\left(\frac{5\pi}{6} - \alpha\right) + \frac{1}{2}\sin(2\alpha + 60°)}$$

3-phase half wave controlled rectifier output voltage Waveforms for two different trigger angles with r load discontinuous load current are shown in figure (24).

Example 3.7

Homework 3.2

Draw the output voltage waveform when $\alpha = 60°$.

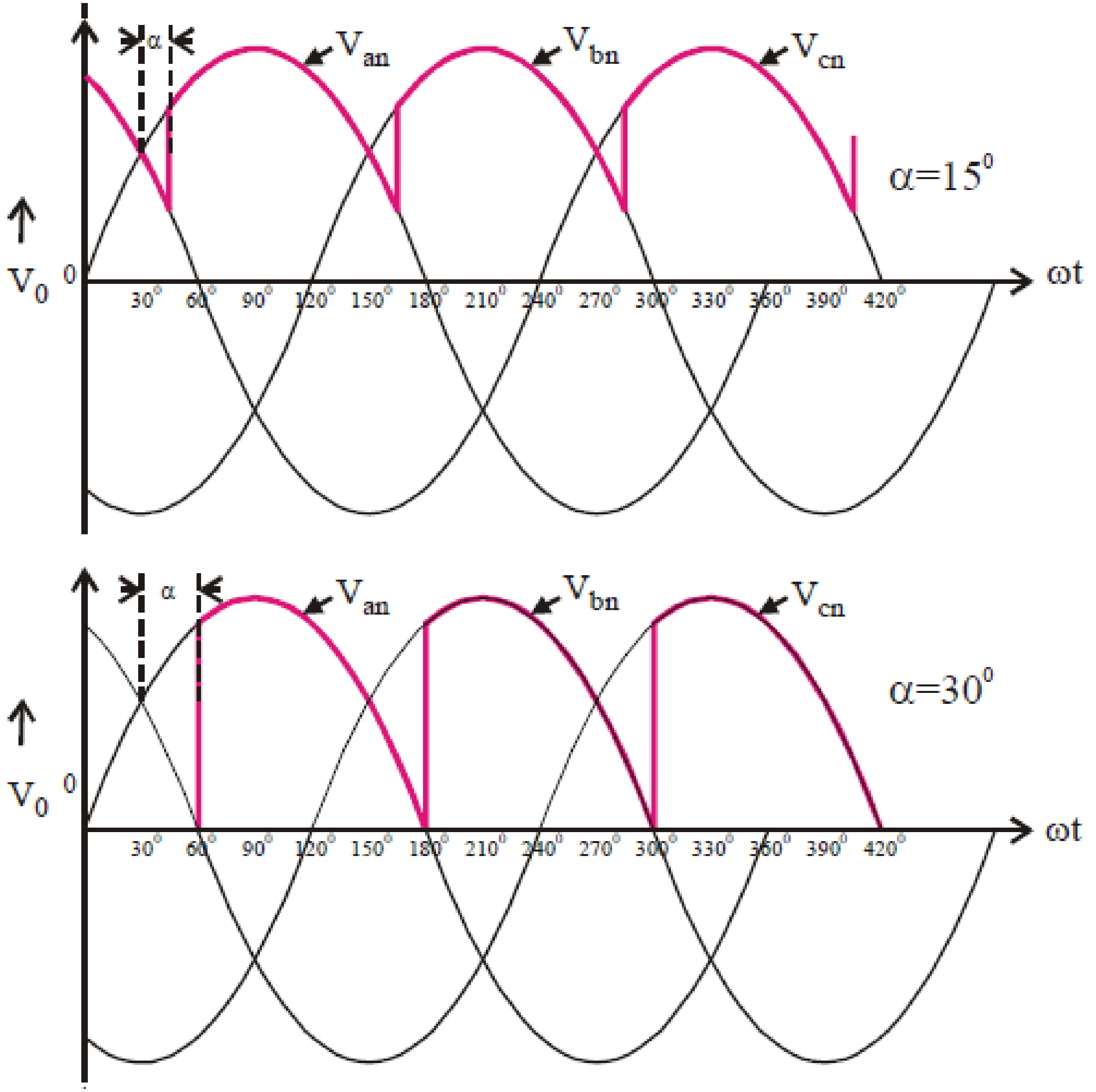


Figure (24) output voltage Waveforms

6- Three-phase semi-controlled rectifier

3-phase semi-converters are three phase half controlled bridge controlled rectifiers which employ three thyristors and three diodes connected in the form of a bridge configuration. Three thyristors are controlled switches which are turned on at appropriate times by applying appropriate gating signals. The three diodes conduct when they are forward biased by the corresponding phase supply voltages.

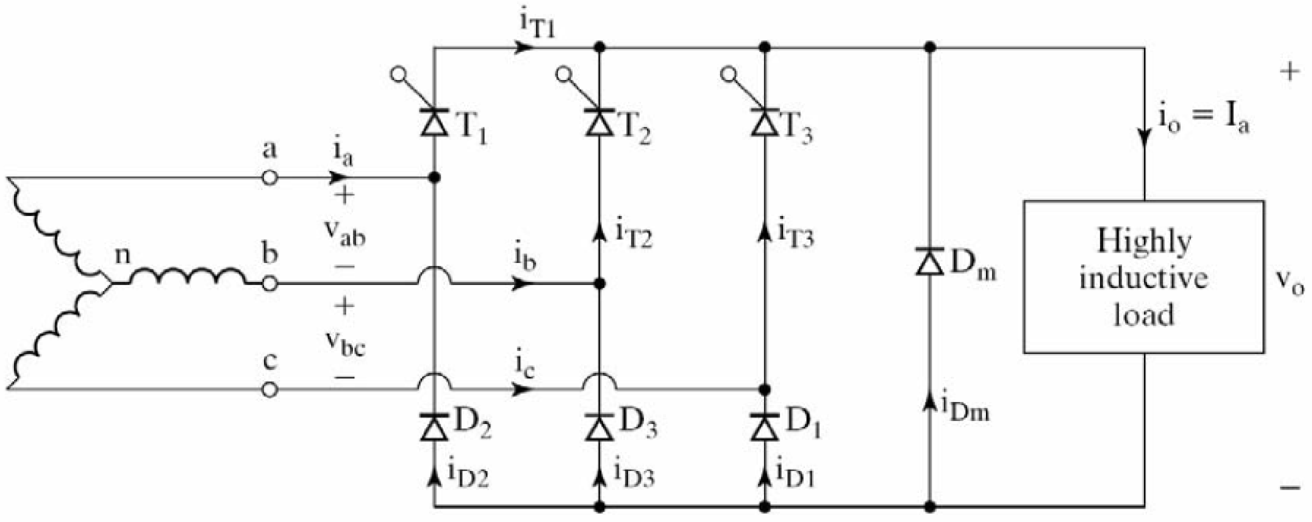


Figure (25) three phase semi-controlled rectifier

3-phase semi-converters are used in industrial power applications up to about 120kW output power level, where single quadrant operation is required. The power factor of 3-phase semi-converter decreases as the trigger angle α increases. The power factor of a 3-phase semi-converter is better than three phase half wave converter. Figure (25) shows a 3-phase semi-converter with a highly inductive load and the load current is assumed to be a constant and continuous load current with negligible ripple.

6.1- The average and rms output voltages for (Resistive load and RL load) If $\boldsymbol{\alpha \geq \frac{\pi}{3}}$ Discontinuous voltage

$$V_{dc} = \frac{3\sqrt{3}V_m}{2\pi}(1+\cos\alpha) \qquad V_{dc} = \frac{3V_{Lm}}{2\pi}(1+\cos\alpha)$$

$$Vo(RMS) = \sqrt{3}V_m\sqrt{\frac{3}{4\pi}\left(\pi-\alpha+\frac{1}{2}\sin 2\alpha\right)}$$

6.2- The average and rms output voltages for (Resistive load and RL load) If $\boldsymbol{\alpha \leq \frac{\pi}{3}}$ Continuous voltage

$$V_{dc} = \frac{3\sqrt{3}V_m}{2\pi}(1+\cos\alpha) \qquad V_{dc} = \frac{3V_{Lm}}{2\pi}(1+\cos\alpha)$$

$$Vo(RMS) = \sqrt{3}V_m\sqrt{\frac{3}{4\pi}\left(\frac{2\pi}{3}+\sqrt{3}\cos^2\alpha\right)}$$

- The current is always continuous in case of RL load
- The current is continuous in case of R load with $\boldsymbol{\alpha \leq \frac{\pi}{3}}$

Homework 3.3:

When the maximum average output voltage occurs at?

The waveforms for a 3-phase semi-converter with $\boldsymbol{\alpha \leq \frac{\pi}{3}}$ is shown in figure (26)

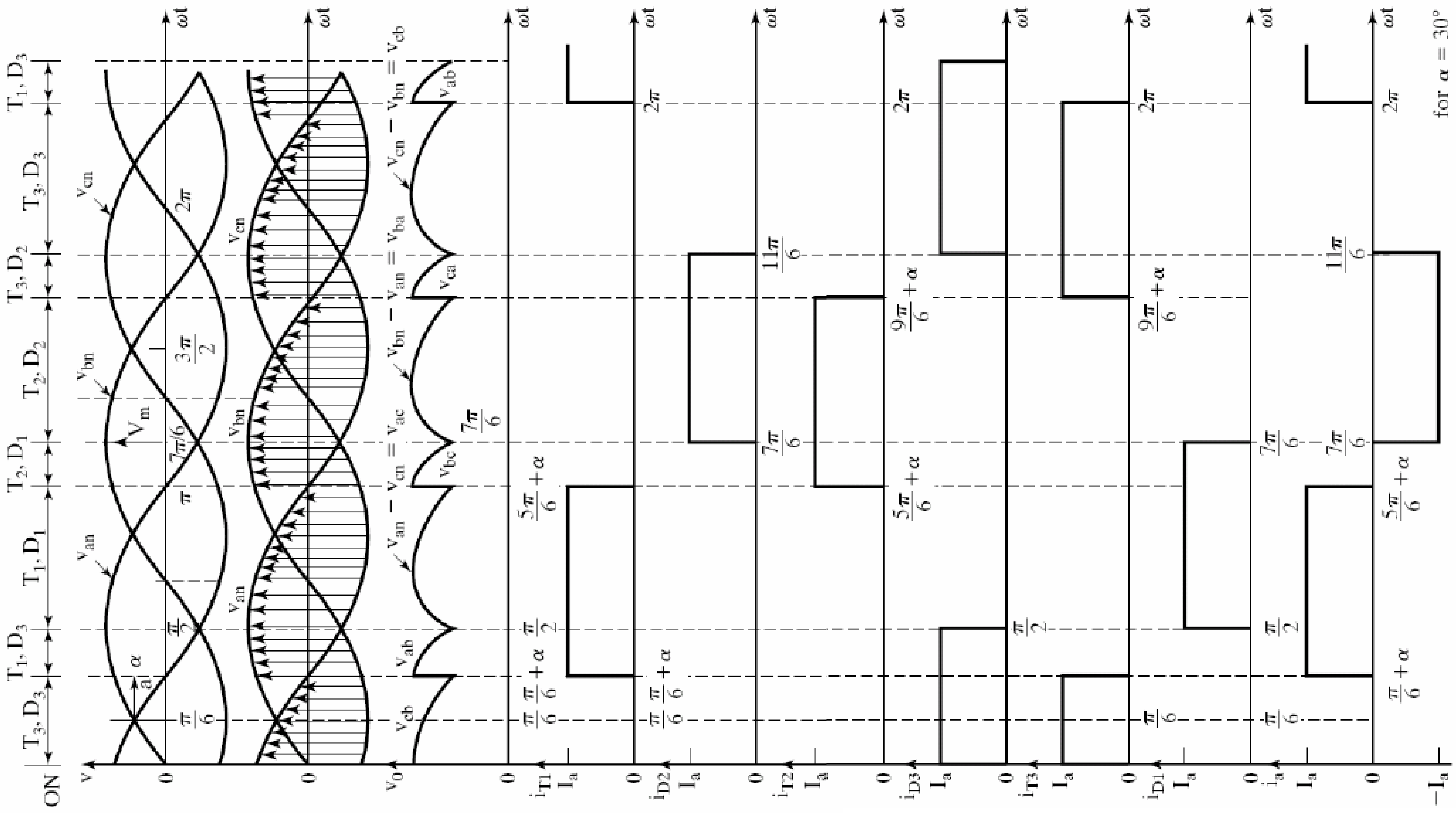


Figure (26) semi-converter output waveforms

7- Three-phase full wave controlled rectifier

Three phase full converter is a fully controlled bridge controlled rectifier using six thyristors connected in the form of a full wave bridge configuration as shown in figure (27). All the six thyristors are controlled switches which are turned on at a appropriate times by applying suitable gate trigger signals. The three phase full converter is extensively used in industrial power applications up to about 120kW output power level, where two quadrant operation is required.

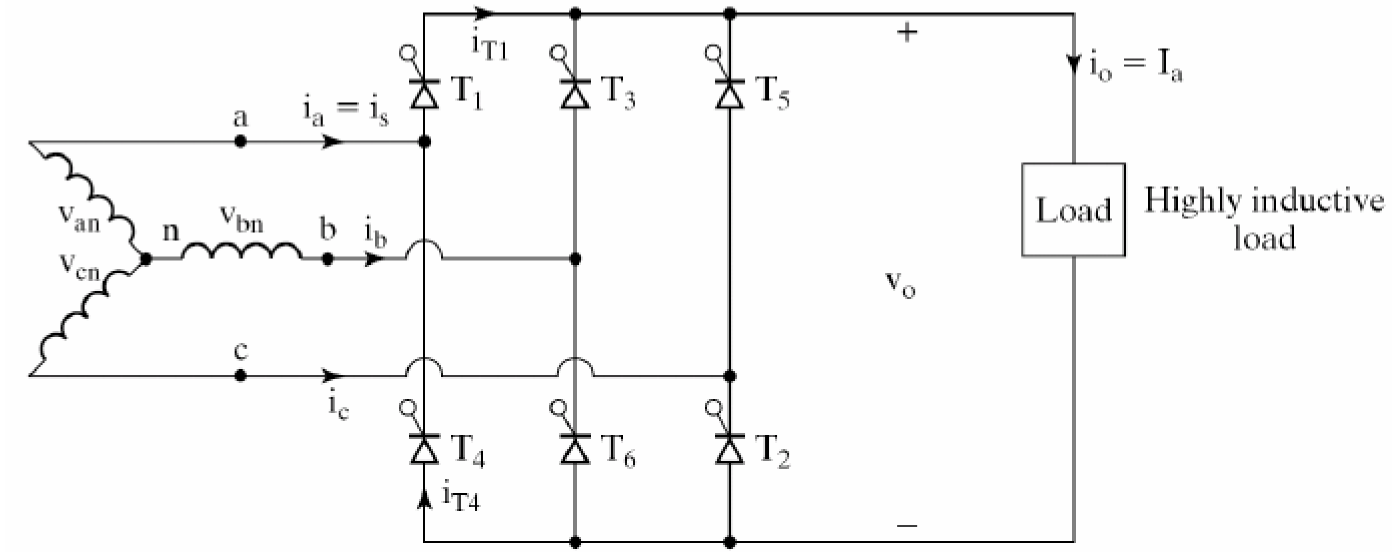
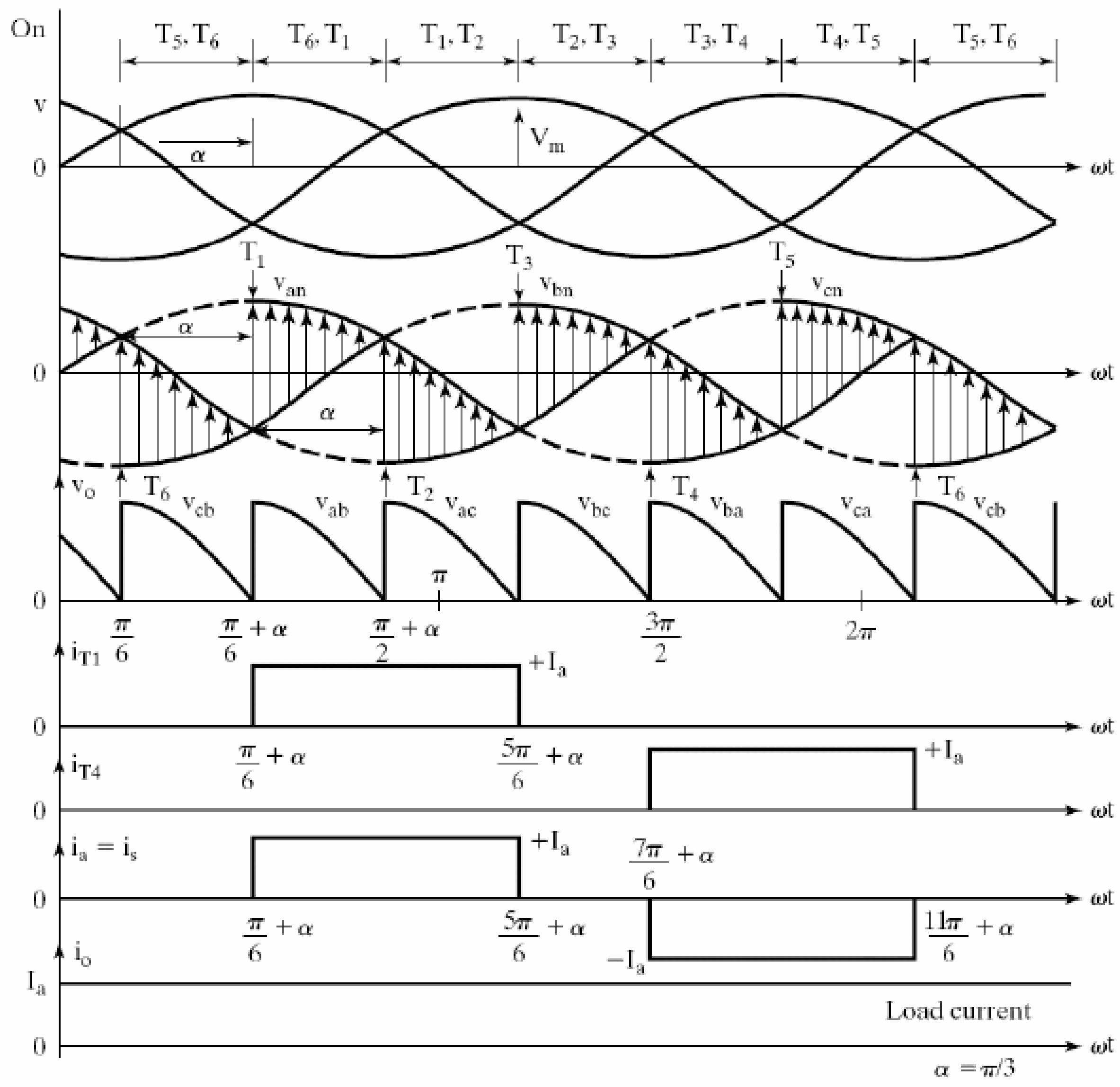
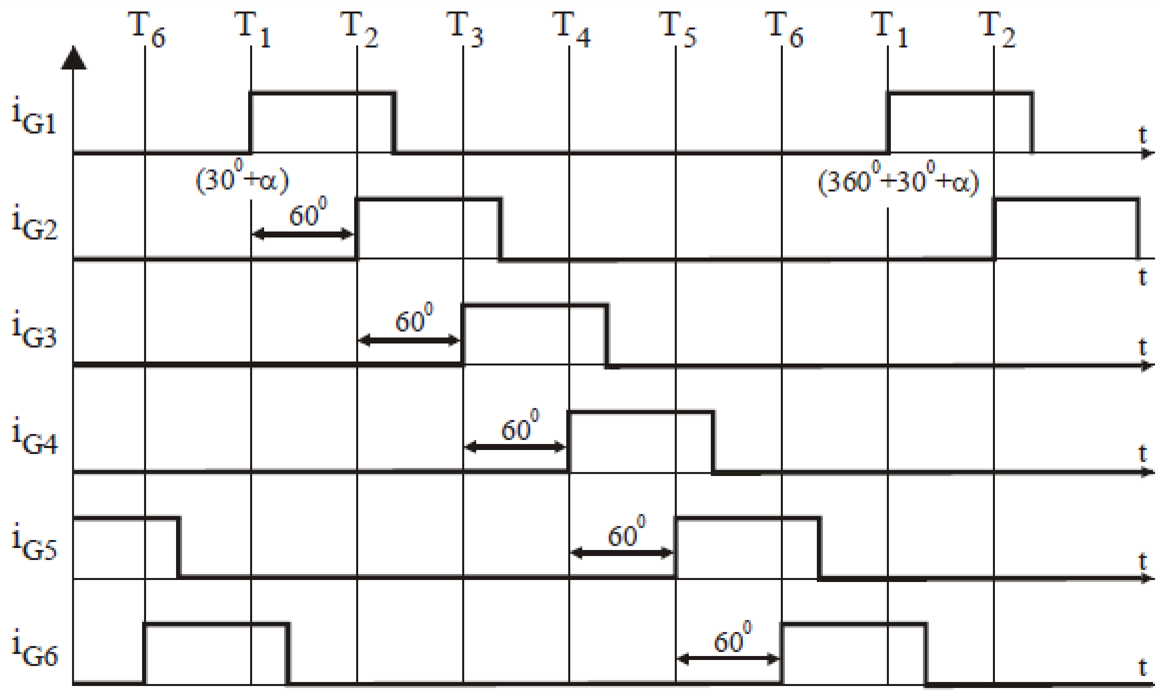


Figure (27) three phase full wave controlled rectifier

The frequency of output ripple voltage is $6fs$ and the filtering requirement is less than that of three phase semi and half wave converters.

The thyristors are numbered in the circuit diagram corresponding to the order in which they are triggered. The trigger sequence (firing sequence) of the thyristors is 12, 23, 34, 45, 56, 61, 12, 23, and so on.




Gating (Control) Signals of 3-phase full converter

The figure (28) shows the waveforms of three phase input supply voltages, output voltage, the thyristor current through *T1* and *T4* , the supply current through the line ‘a’.

7.1- The average and rms output voltages for continuous and constant load current (RL load)
For any α

$$V_{dc} = \frac{3\sqrt{3}V_m}{\pi}\cos\alpha \qquad V_{dc} = \frac{3V_{Lm}}{\pi}\cos\alpha$$

$$Vo(RMS) = \sqrt{3}V_m\sqrt{\frac{1}{2} + \frac{3\sqrt{3}}{4\pi}\cos 2\alpha}$$

The rms value of source current for any phase $Is = Io\sqrt{\frac{2}{3}}$

Where Io is a constant output current

7.2- The average and rms output voltages for discontinuous load current (Resistive load only)

a) For $\alpha \le 60°$

$$V_{dc} = \frac{3\sqrt{3}V_m}{\pi}\cos\alpha \qquad V_{dc} = \frac{3V_{Lm}}{\pi}\cos\alpha$$

$$Vo(RMS) = \sqrt{3}V_m\sqrt{\frac{1}{2} + \frac{3\sqrt{3}}{4\pi}\cos 2\alpha}$$

a) For $\alpha > 60°$

$$V_{dc} = \frac{6V_m}{2\pi}[1 + \cos(60° + \alpha)]$$

Example 3.8 , Example 3.9 , Example 3.10

Homework sheet no. 2

Chapter 4
DC to DC converters

Dc-dc converters are power electronic circuits that convert a dc voltage to a different dc voltage level, often providing a regulated output. The circuits described in this chapter are classified as switched-mode dc-dc converters, also called switching power supplies or switchers.

1- Linear voltage regulators

Before we discuss switched-mode converters, it is useful to review the motivation for an alternative to linear dc-dc converters. One method of converting a dc voltage to a lower dc voltage is a simple circuit as shown in Figure (29). The output voltage is $V_o = I_L R_L$

Where the load current is controlled by the transistor. By adjusting the transistor base current, the output voltage may be controlled over a range of 0 to roughly Vs.

The base current can be adjusted to compensate for variations in the supply voltage or the load, thus regulating the output. This type of circuit is called a linear dc-dc converter or a linear regulator because the transistor operates in the linear region, rather than in the saturation or cut-off regions. The transistor in effect operates as a variable resistance. While this may be a simple way of converting a dc supply voltage to a lower dc voltage and regulating the output, the low efficiency of this circuit is a serious drawback for power applications.

The power absorbed by the load is $V_o I_L$, and the power absorbed by the transistor is $V_{CE} I_L$, assuming a small base current. The power loss in the transistor makes this circuit inefficient. For example, if the output voltage is one-quarter of the input voltage, the load resistor absorbs one-quarter of the source power, which is an efficiency of 25 percent. The transistor absorbs the other 75 percent of the power supplied by the source. Lower output voltages result in even lower efficiencies. Therefore, the linear voltage regulator is suitable only for low-power applications.

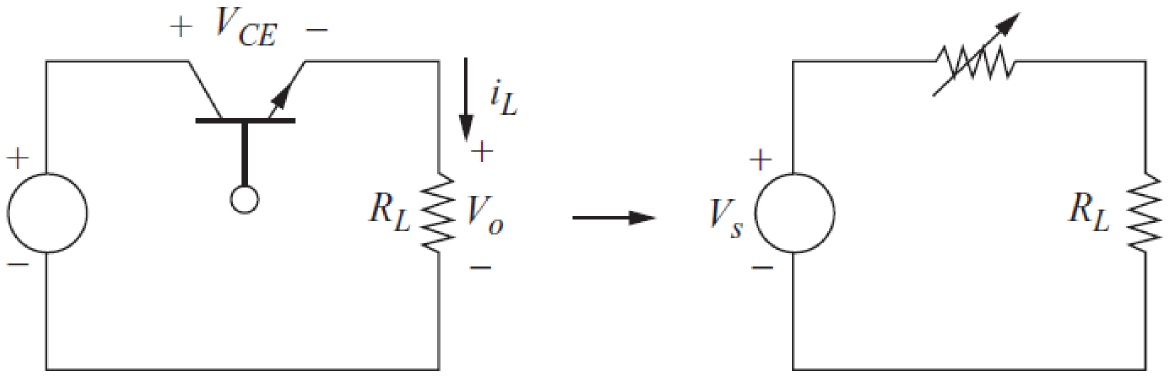


Figure (29) A basic linear regulator.

2- A basic switching converter

An efficient alternative to the linear regulator is the switching converter. In a switching converter circuit, the transistor operates as an electronic switch by being completely on or completely off (saturation or cut-off for a BJT or the triode and cut-off regions of a MOSFET). This circuit is also known as a dc chopper. Assuming the switch is ideal in Figure (30), the output is the same as the input when the switch is closed, and the output is zero when the switch is open. Periodic opening and closing of the switch results in the pulse output shown in Figure (30-c). The average or dc component of the output voltage is

$$V_o = \frac{1}{T}\int_0^T V_o(t)\,dt = \frac{1}{T}\int_0^{DT} V_s dt = V_s D$$

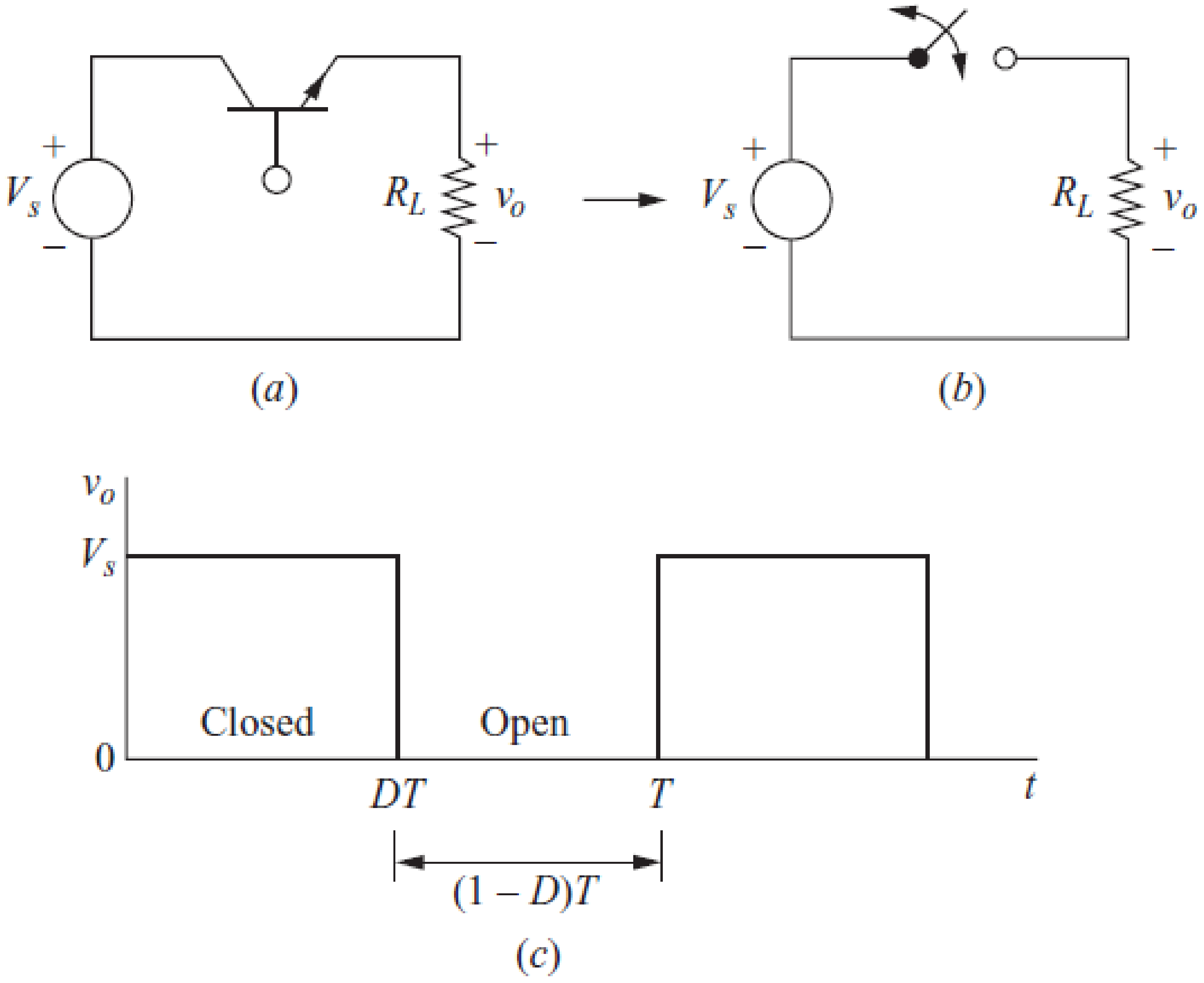


Figure (30) (*a*) A basic dc-dc switching converter; (*b*) Switching equivalent; (*c*) Output voltage.

The dc component of the output voltage is controlled by adjusting the duty ratio D, which is the fraction of the switching period that the switch is closed where f is the switching frequency.

$$D \equiv \frac{t_{on}}{t_{on} + t_{off}} = \frac{t_{on}}{T} = t_{on} f$$

The dc component of the output voltage will be less than or equal to the input voltage for this circuit. The power absorbed by the ideal switch is zero. When the switch is open, there is no current in it; when the switch is closed, there is no voltage across it. Therefore, all power is absorbed by the load, and the energy efficiency is 100 percent. Losses will occur in a real switch because the voltage across it will not be zero when it is on, and the switch must pass through the linear region when making a transition from one state to the other.

3- The buck (step-down) converter

Controlling the dc component of a pulsed output voltage of the type in Figure (30-c) may be sufficient for some applications, such as controlling the speed of a dc motor, but often the objective is to produce an output that is purely dc. One way of obtaining a dc output from the circuit of Figure (30-a) is to insert a low-pass filter after the switch. Figure (31-a) shows an LC low-pass filter added to the basic converter. The diode provides a path for the inductor current when the switch is opened and is reverse-biased when the switch is closed. This circuit is called a buck converter or a step-down converter because the output voltage is less than the input.

If the low-pass filter is ideal, the output voltage is the average of the input voltage to the filter. The input to the filter, V_x in Figure (31-a), is V_s when the switch is closed and is zero when the switch is open, provided that the inductor current remains positive, keeping the diode on. If the switch is closed periodically at a duty ratio D, the average voltage at the filter input is V_sD.

This analysis assumes that the diode remains forward-biased for the entire time when the switch is open, implying that the inductor current remains positive. An inductor current that remains positive throughout the switching period is known as continuous current. Conversely, discontinuous current is characterized by the inductor current's returning to zero during each period.

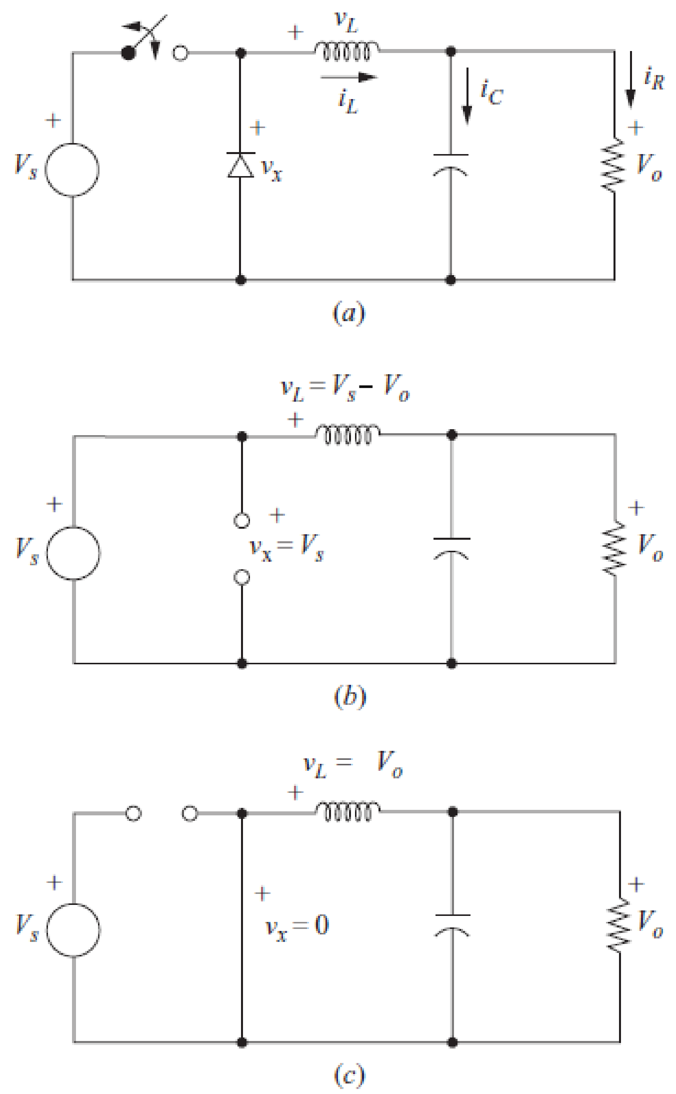


Figure (31) (a) Buck dc-dc converter; (b) Equivalent circuit for the switch closed; (c) Equivalent circuit for the switch open.

3.1- Analysis for the Switch Closed

When the switch is closed in the buck converter circuit of Figure (31-a), the diode is reverse biased and Figure (31-b) is an equivalent circuit. The voltage across the inductor is

$$V_L = V_s - V_o = L\frac{di_L}{dt}$$

$$\frac{di_L}{dt} = \frac{V_s - V_o}{L}$$

Since the derivative of the current is a positive constant, the current increases linearly as shown in Figure (32-b). The change in current while the switch is closed is computed by modifying the preceding equation.

$$\frac{di_L}{dt} = \frac{\Delta i_L}{\Delta t} = \frac{\Delta i_L}{DT} = \frac{V_s - V_o}{L} \Longrightarrow (\Delta i_L)closed = \left(\frac{V_s - V_o}{L}\right)DT$$

$$DT = \frac{t_{on}}{t_{on} + t_{off}}T = \frac{t_{on}}{T}T = t_{on}$$

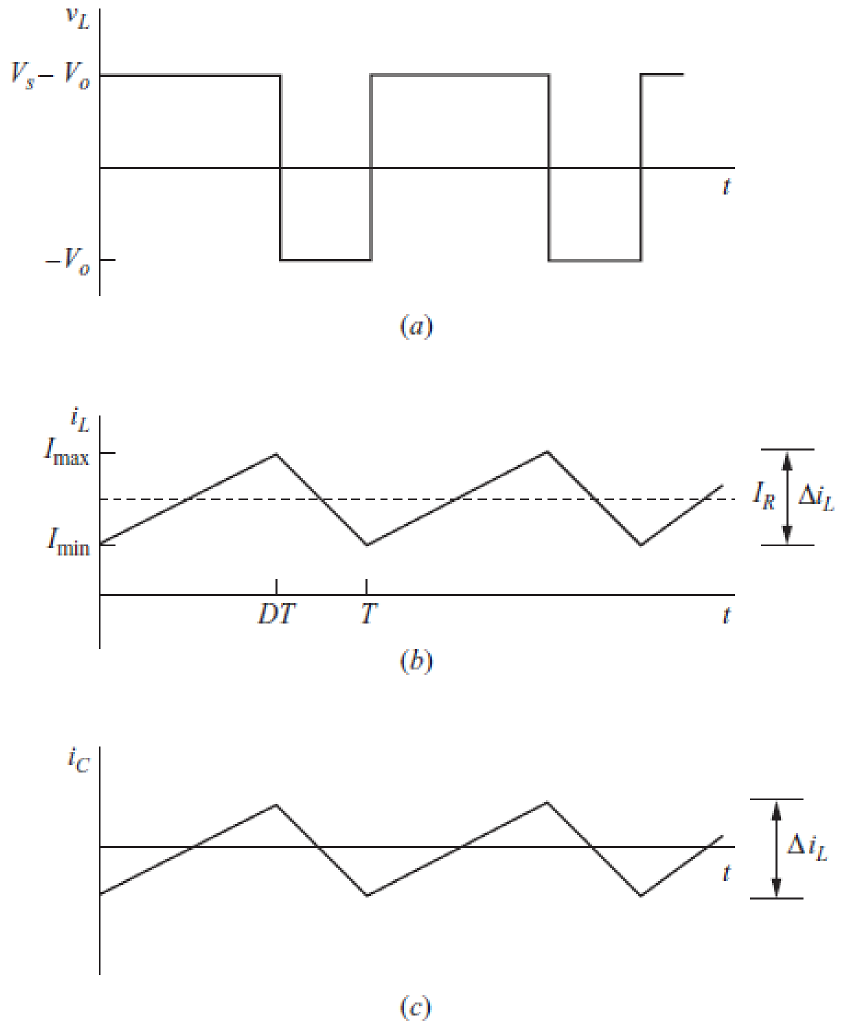


Figure (32) Buck converter waveforms: (a) Inductor voltage; (b) Inductor current; (c) Capacitor current..

3.2- Analysis for the Switch Open

When the switch is open, the diode becomes forward-biased to carry the inductor current and the equivalent circuit of Figure (31-c) applies. The voltage across the inductor when the switch is open is

$$V_L = -V_o = L\frac{di_L}{dt}$$

$$\frac{di_L}{dt} = \frac{-V_o}{L}$$

The derivative of current in the inductor is a negative constant, and the current decreases linearly as shown in Figure (32-b). The change in inductor current when the switch is open is

$$\frac{di_L}{dt} = \frac{\Delta i_L}{(1-D)T} = \frac{-V_o}{L} \Rightarrow (\Delta i_L)open = -\left(\frac{V_o}{L}\right)(1-D)T$$

Steady-state operation requires that the inductor current at the end of the switching cycle be the same as that at the beginning, meaning that the net change in inductor current over one period is zero. This requires

$$(\Delta i_L)closed + (\Delta i_L)open = 0 \Rightarrow \left(\frac{V_s - V_o}{L}\right)DT - \left(\frac{V_o}{L}\right)(1-D)T = 0$$

Solving for V_o $\quad V_o = V_s D \qquad V_o av = V_s D \qquad V_o rms = V_s\sqrt{D}$

The buck converter produces an output voltage that is less than or equal to the input.

To determine the value of inductance for a specified peak-to-peak inductor current for continuous-current operation:

$$\Delta i_L = \left(\frac{V_s - V_o}{L}\right) DT = \left(\frac{V_s - V_o}{Lf}\right) D = \frac{V_o(1-D)}{Lf}$$

$$L = \left(\frac{V_s - V_o}{\Delta i_L f}\right) D = \frac{V_o(1-D)}{\Delta i_L f}$$

Where f is the switching frequency.
The minimum inductance L_{min}required for continuous current:

$$L_{min} = \frac{(1-D)R}{2f}$$

In practice, a value of inductance greater than L_{min} is desirable to ensure continuous current.

The average inductor current must be the same as the average current in the load resistor, since the average capacitor current must be zero for steady-state operation: $I_L = I_R = \frac{V_o}{R}$

Since the change in inductor current is known, the maximum and minimum values of the inductor current are computed as:

$$I_{max} = I_L + \frac{\Delta i_L}{2} = \frac{V_o}{R} + \frac{\left(\frac{V_s - V_o}{L}\right) DT}{2} = \frac{V_o}{R} + \frac{1}{2}\left[\frac{V_o}{L}(1-D)T\right]$$

$$I_{max} = V_o\left(\frac{1}{R} + \frac{1-D}{2Lf}\right)$$

$$I_{min} = I_L - \frac{\Delta i_L}{2} = \frac{V_o}{R} - \frac{\left(\frac{V_s - V_o}{L}\right) DT}{2} = \frac{V_o}{R} - \frac{1}{2}\left[\frac{V_o}{L}(1-D)T\right]$$

$$I_{min} = V_o\left(\frac{1}{R} - \frac{1-D}{2Lf}\right)$$

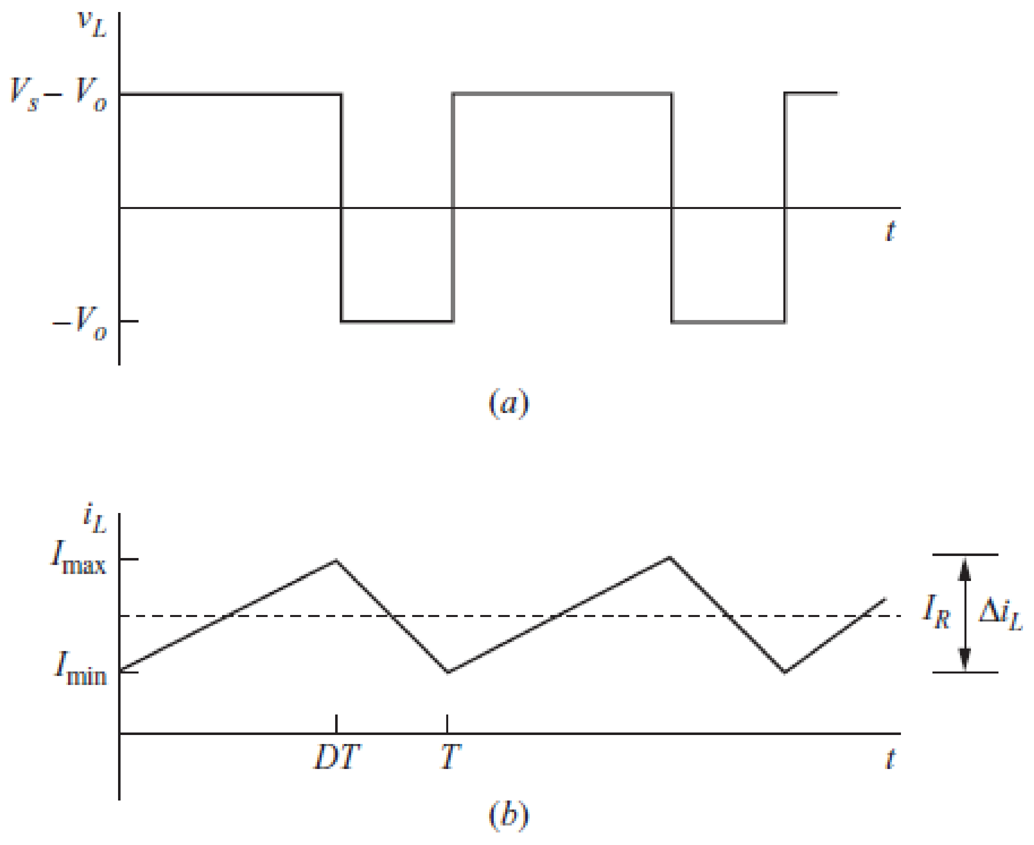


Figure (32-a,b)

3.3- Output Voltage Ripple

In the preceding analysis, the capacitor was assumed to be very large to keep the output voltage constant. In practice, the output voltage cannot be kept perfectly constant with a finite capacitance. The variation in output voltage, or ripple, is computed from the voltage-current relationship of the capacitor. The current in the capacitor is

$i_C = i_L - i_R$ shown in figure (33-a)

While the capacitor current is positive, the capacitor is charging. From the definition of capacitance,

$Q = CV_o \quad \Delta Q = C\Delta V_o \quad \Delta V_o = \dfrac{\Delta Q}{C}$

The change in charge ΔQ is the area of the triangle above the time axis $\Delta Q = \frac{1}{2}\left(\frac{T}{2}\right)\left(\frac{\Delta i_L}{2}\right) = \frac{T\Delta i_L}{8}$

$$\therefore \ \Delta V_o = \frac{T\Delta i_L}{8C}$$

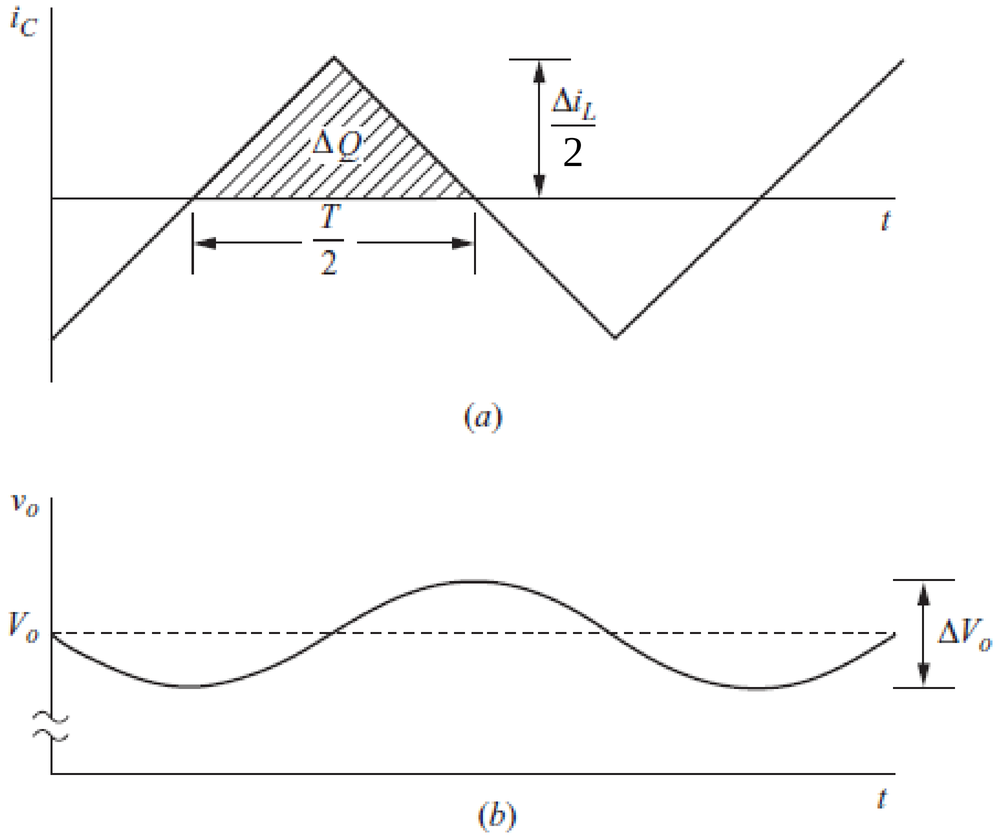


Figure (33) Buck converter waveforms.
(a) Capacitor current;
(b) Capacitor ripple voltage.

$$\Delta V_o = \frac{T\Delta i_L}{8C}$$

$$\because \ \Delta i_L = \left(\frac{V_s - V_o}{L}\right) DT = \frac{V_o}{L}(1-D)T \qquad \therefore \ \frac{\Delta V_o}{V_o} = \frac{1-D}{8LCf^2}$$

Example 4.1

Homework 4.1
You may need this

$$I_{L,\mathrm{rms}} = \sqrt{I_L^2 + \left(\frac{\Delta i_L/2}{\sqrt{3}}\right)^2}$$

And Let the switching frequency arbitrarily be 40 kHz, which is well above the audio range and is low enough to keep switching losses small.

4- The boost converter

The boost converter is shown in Figure (34). This is another switching converter that operates by periodically opening and closing an electronic switch. It is called a boost converter because the output voltage is larger than the input.

- Steady-state conditions exist.
- The switching period is T, and the switch is closed for time DT and open for $(1-D)T$ as shown in figure (35-a)

- $T - DT = (1-D)T$
- The inductor current is continuous (always positive).
- The capacitor is very large, and the output voltage is held constant at voltage Vo.

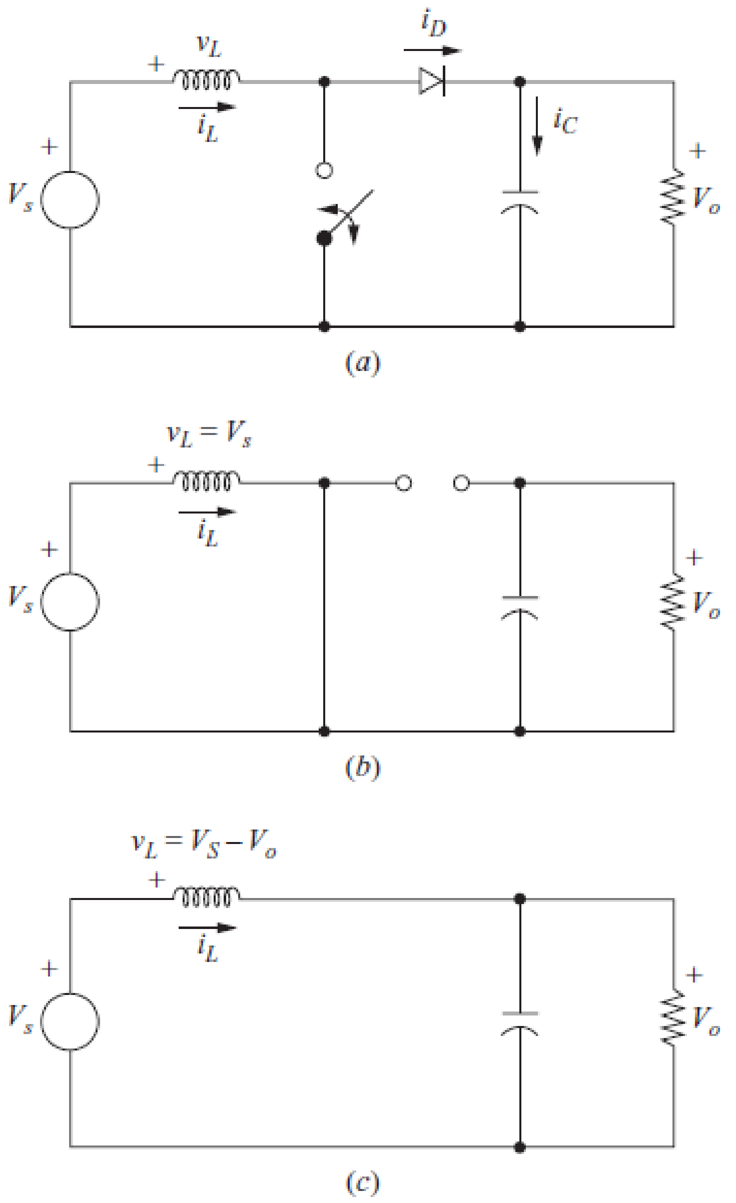


Figure (34) The boost converter. (*a*) Circuit; (*b*) Equivalent circuit for the switch closed; (*c*) Equivalent circuit for the switch open.

4.1- Analysis for the Switch Closed

When the switch is closed, the diode is reverse biased. Kirchhoff's voltage law around the path containing the source, inductor, and closed switch is

$$V_L = V_S = L\frac{di_L}{dt} \Longrightarrow \frac{di_L}{dt} = \frac{V_S}{L}$$

The rate of change of current is a constant, so the current increases linearly while the switch is closed, as shown in Figure (35-b).

The change in inductor current is computed from

$$\frac{\Delta i_L}{\Delta t} = \frac{\Delta i_L}{DT} = \frac{V_S}{L}$$

$$\therefore (\Delta i_L)closed = \frac{V_S DT}{L}$$

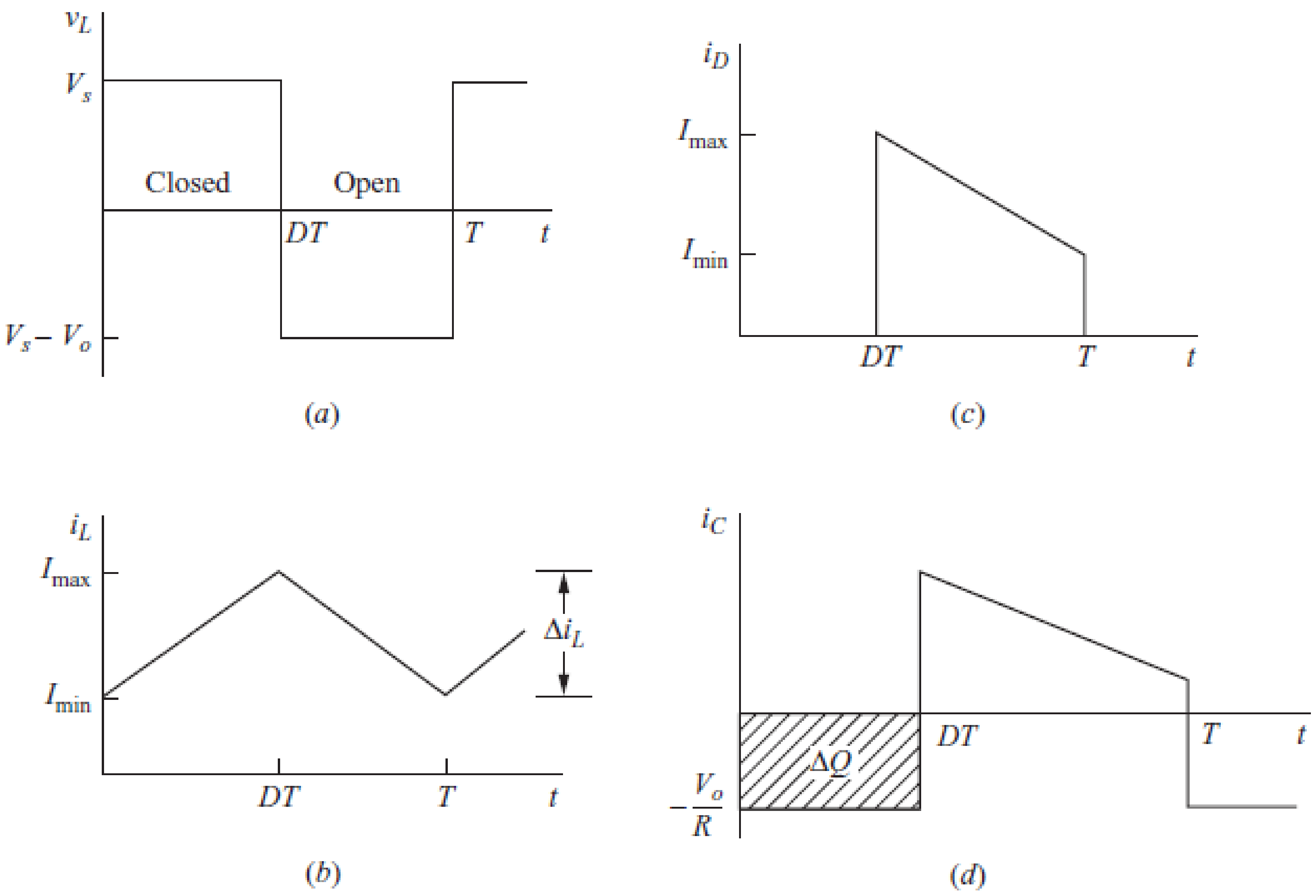


Figure (35) Boost converter waveforms.
(*a*) Inductor voltage; (*b*) Inductor current;
(*c*) Diode current; (*d*) Capacitor current.

4.2- Analysis for the Switch Open When the switch is opened, the inductor current cannot change instantaneously, so the diode becomes forward-biased to provide a path for inductor current. Assuming that the output voltage *Vo* is a constant, the voltage across the inductor is

$$V_L = V_S - V_O = L\frac{di_L}{dt}$$

$$\frac{di_L}{dt} = \frac{V_S - V_O}{L}$$

The rate of change of inductor current is a constant, so the current must change linearly while the switch is open. The change in inductor current while the switch is open is

$$\frac{\Delta i_L}{\Delta t} = \frac{\Delta i_L}{(1-D)T} = \frac{V_S - V_O}{L}$$

The rate of change of inductor current is a constant, so the current must change linearly while the switch is open. The change in inductor current while the switch is open is

$$\frac{\Delta i_L}{\Delta t} = \frac{\Delta i_L}{(1-D)T} = \frac{V_S - V_O}{L}$$

$$(\Delta i_L)open = \frac{(V_S - V_O)(1-D)T}{L}$$

For steady-state operation, the net change in inductor current must be zero.

$$(\Delta i_L)closed + (\Delta i_L)open = 0$$

$$\frac{V_S DT}{L} + \frac{(V_S - V_O)(1-D)T}{L} = 0$$

Solving for Vo,

$$V_S(D + 1 - D) - V_O(1 - D) = 0$$

$$V_O = \frac{V_S}{1 - D}$$

The average current in the inductor is determined by recognizing that the average power supplied by the source must be the same as the average power absorbed by the load resistor. Output power is

$$P_O = \frac{V_O{}^2}{R} = V_O I_O$$ and input power is $V_S I_S = V_S I_L$.

Equating input and output powers

$$V_S I_L = \frac{V_O{}^2}{R} = \frac{\left[\frac{V_S}{(1-D)}\right]^2}{R} = \frac{V_S{}^2}{(1-D)^2 R}$$ By solving for average inductor current and making various substitutions,

IL can be expressed as $$I_L = \frac{V_S}{(1-D)^2 R} = \frac{V_O{}^2}{V_S R} = \frac{V_O I_O}{V_S}$$

Maximum and minimum inductor currents are determined by using the average value and the change in current

$$I_{max} = I_L + \frac{\Delta i_L}{2} = \frac{V_S}{(1-D)^2 R} + \frac{V_S DT}{2L}$$

$$I_{min} = I_L - \frac{\Delta i_L}{2} = \frac{V_S}{(1-D)^2 R} - \frac{V_S DT}{2L}$$

From a design perspective, it is useful to express L in terms of a desired Δi_L,

$$L = \frac{V_S DT}{\Delta i_L} = \frac{V_S D}{\Delta i_L f}$$

The minimum combination of inductance and switching frequency for continuous current in the boost converter is therefore

$$L_{min} = \frac{D(1-D)^2 R}{2f}$$

4.3- Output Voltage Ripple

The preceding equations were developed on the assumption that the output voltage was a constant, implying an infinite capacitance. In practice, a finite capacitance will result in some fluctuation in output voltage, or ripple. The peak-to-peak output voltage ripple can be calculated from the capacitor current waveform, shown in Figure (35-d). The change in capacitor charge can be calculated from

$$|\Delta Q| = \left(\frac{V_O}{R}\right) DT = C\Delta V_O$$

$$\Delta V_O = \frac{V_O DT}{CR} = \frac{V_O D}{CRf}$$

$$\frac{\Delta V_O}{V_O} = \frac{D}{RCf}$$

where f is the switching frequency. Alternatively, expressing capacitance in terms of output voltage ripple yields $C = \dfrac{D}{R\left(\frac{\Delta V_O}{V_O}\right)f}$

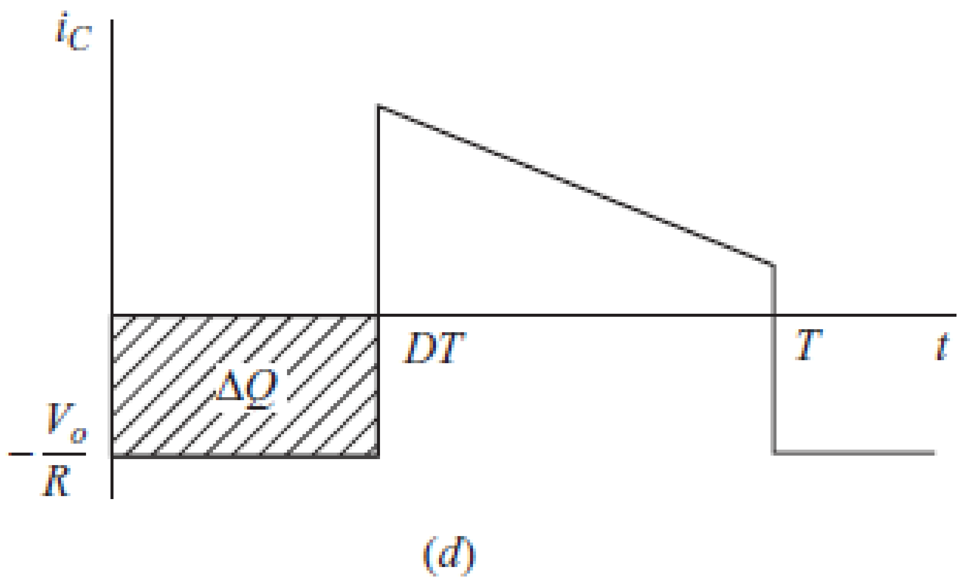


Figure (35-d) Capacitor current.

Example 4.2

Homework 4.2

5- The buck-boost converter

Another basic switched-mode converter is the buck-boost converter shown in Figure (36). The output voltage of the buck-boost converter can be either higher or lower than the input voltage.

- The circuit is operating in the steady state.
- The inductor current is continuous.
- The capacitor is large enough to assume a constant output voltage.
- The switch is closed for time DT and open for $(1 - D)T$.
- The components are ideal.

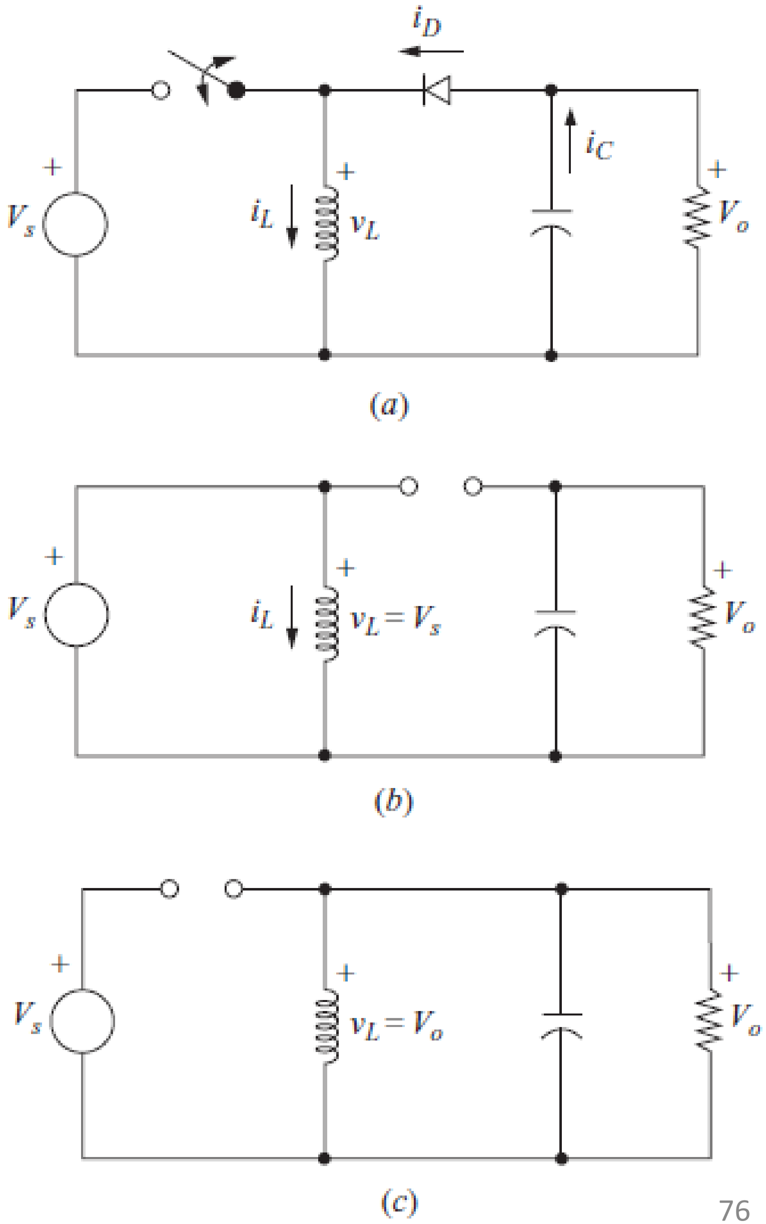


Figure (36) Buck-boost converter. (*a*) Circuit;
(*b*) Equivalent circuit for the switch closed;
(*c*) Equivalent circuit for the switch open.

5.1- Analysis for the Switch Closed

When the switch is closed, the voltage across the inductor is

$$V_L = V_S = L\frac{di_L}{dt} \qquad \frac{di_L}{dt} = \frac{V_S}{L}$$

The rate of change of inductor current is a constant, indicating a linearly increasing inductor current. The preceding equation can be expressed as

$$\frac{\Delta i_L}{\Delta t} = \frac{\Delta i_L}{DT} = \frac{V_S}{L}$$ Solving for Δi_L when the switch is closed gives $$(\Delta i_L)closed = \frac{V_S DT}{L}$$

5.2- Analysis for the Switch Open

When the switch is open, the current in the inductor cannot change instantaneously, resulting in a forward-biased diode and current into the resistor and capacitor. In this condition, the voltage across the inductor is

$$V_L = V_O = L\frac{di_L}{dt}$$

$$\frac{di_L}{dt} = \frac{V_o}{L} \Longrightarrow \frac{\Delta i_L}{\Delta t} = \frac{\Delta i_L}{(1-D)T} = \frac{V_O}{L}$$

$$(\Delta i_L)open = \frac{V_O(1-D)T}{L}$$

For steady-state operation, the net change in inductor current must be zero over one period.

$$(\Delta i_L)closed + (\Delta i_L)open = 0$$

$$\frac{V_S DT}{L} + \frac{V_O(1-D)T}{L} = 0$$

Solving for Vo

$$V_O = -V_S\left(\frac{D}{1-D}\right)$$

The required duty ratio for specified input and output voltages can be expressed as

$$D = \frac{|V_O|}{V_S + |V_O|}$$

Figure (37) shows Buck-boost converter waveforms.

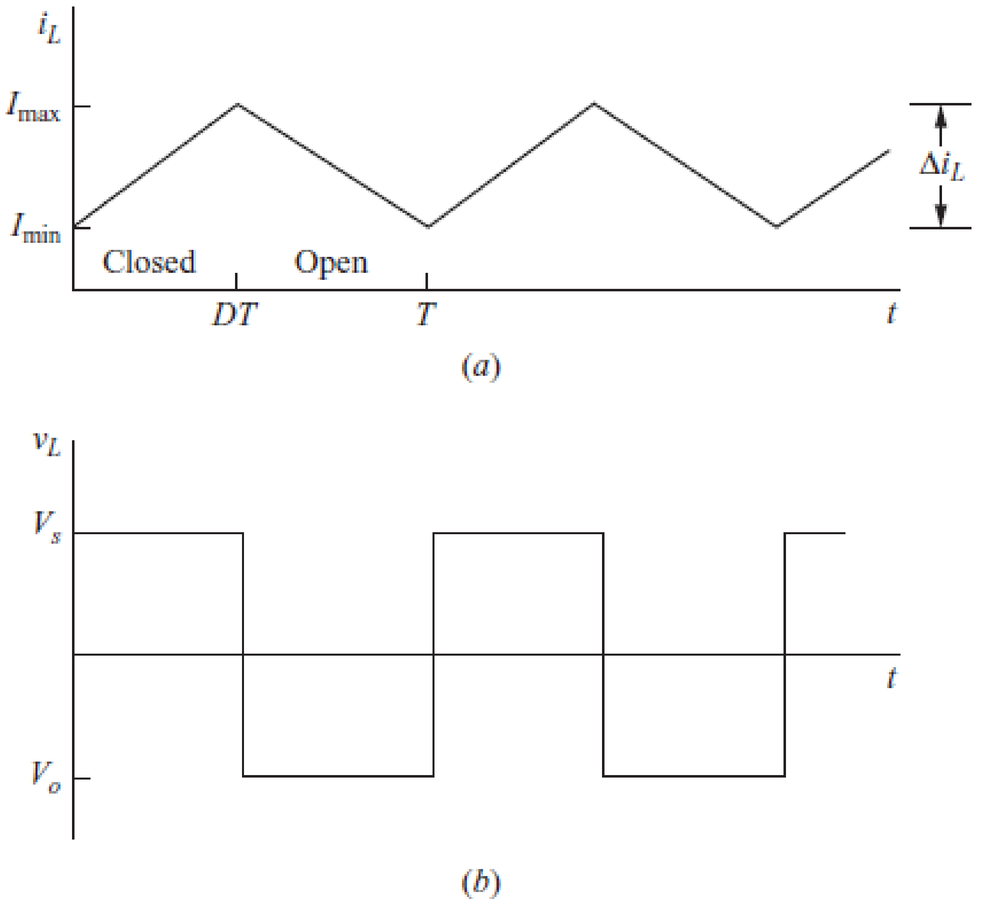


Figure (37) Buck-boost converter waveforms. (*a*) Inductor current; (*b*) Inductor voltage

Note that the source is never connected directly to the load in the buck-boost converter. Energy is stored in the inductor when the switch is closed and transferred to the load when the switch is open. Hence, the buck-boost converter is also referred to as an indirect converter.

Power absorbed by the load must be the same as that supplied by the source, where

$$P_O = \frac{V_O{}^2}{R} \qquad P_S = V_S I_S \qquad \frac{V_O{}^2}{R} = V_S I_S$$

Average source current is related to average inductor current by

$$I_S = I_L D \qquad \frac{V_O{}^2}{R} = V_S I_L D$$

$$I_L = \frac{V_O{}^2}{V_S R D} = \frac{P_O}{V_S D} = \frac{V_S D}{R(1-D)^2}$$

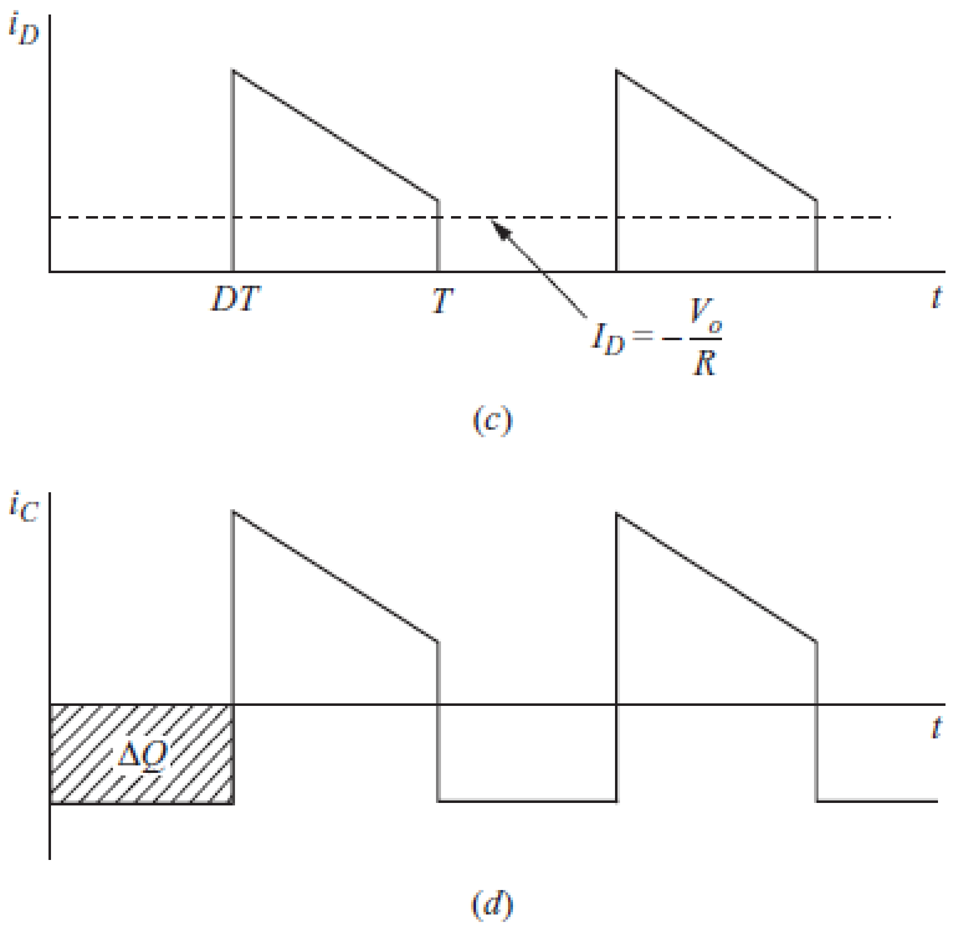


Figure (37) Buck-boost converter waveforms. (c) Diode current; (d) Capacitor current.

Maximum and minimum inductor currents are determined using

$$I_{max} = I_L + \frac{\Delta i_L}{2} = \frac{V_S D}{R(1-D)^2} + \frac{V_S DT}{2L}$$

$$I_{min} = I_L + \frac{\Delta i_L}{2} = \frac{V_S D}{R(1-D)^2} - \frac{V_S DT}{2L}$$

The minimum combination of inductance and switching frequency for continuous current in the buck-boost converter is therefore

$$L_{min} = \frac{(1-D)^2 R}{2f}$$

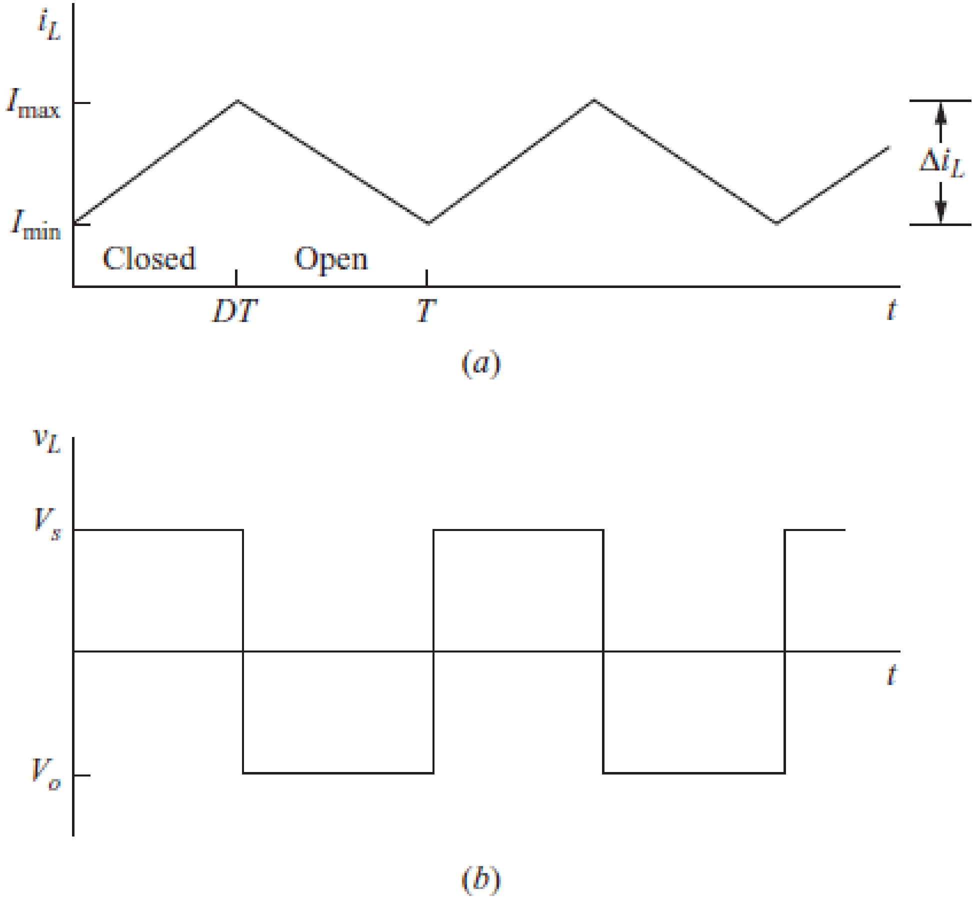


Figure (37) Buck-boost converter waveforms. (*a*) Inductor current; (*b*) Inductor voltage

5.3- Output Voltage Ripple

The output voltage ripple for the buck-boost converter is computed from the capacitor current waveform of Figure (35-d).

$$|\Delta Q| = \left(\frac{V_O}{R}\right) DT = C \Delta V_O$$

$$\Delta V_O = \frac{V_O DT}{RC} = \frac{V_O D}{RCf}$$

$$\frac{\Delta V_O}{V_O} = \frac{D}{RCf}$$

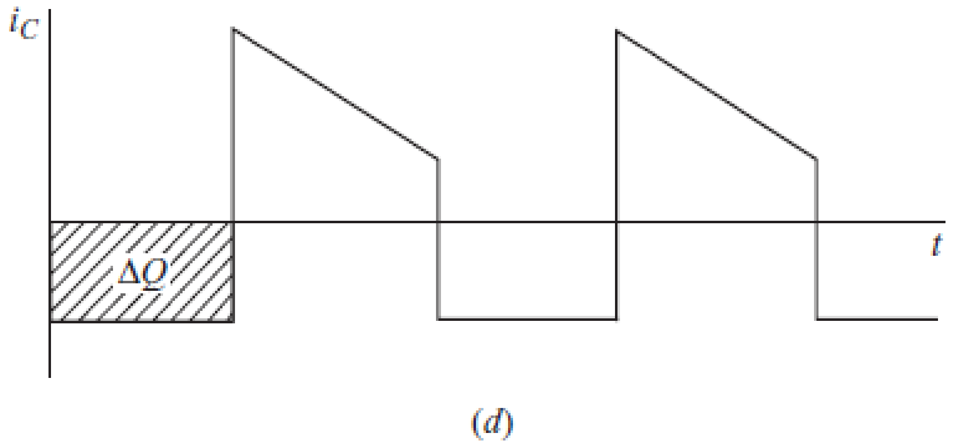


Figure (37) Buck-boost converter waveforms. (d) Capacitor current.

There is something called the equivalent series resistance of the capacitor (ESR) which is exist in all the three types of converters. It contributes significantly to the output ripple voltage.

$$\Delta V_O ESR = \Delta i_C r_C = I_{L\,max} r_C$$

This formula can be used for the three converters

Example 4.3

Chapter 5
AC converters

5.1- Transformer:

The purpose of a transformer is to change an electric system quantity (e.g., voltage or current) from one level to another. A transformer is made up of two or more conductors wound around a single magnetic core, usually iron. The wound conductors, usually copper, are called windings. Figure (38) shows an example of a three phase transformer.

The primary winding is electrically connected to the power source. The secondary winding is electrically connected to the energy output or load side. There is no electrical connection between the primary and secondary windings.

Figure (38) Three phase Transformer

There are three main types of a transformer:

1- Step-up transformer

2- Step-Down transformer

3- Auto Transformer

Each one of these transformers converts the ac input voltage from one value to another without changing its frequency. Therefore, a transformer can only changes the amplitude of an ac voltage and keeps the width of the waveform unaltered. Figure (39) explains this very clearly.

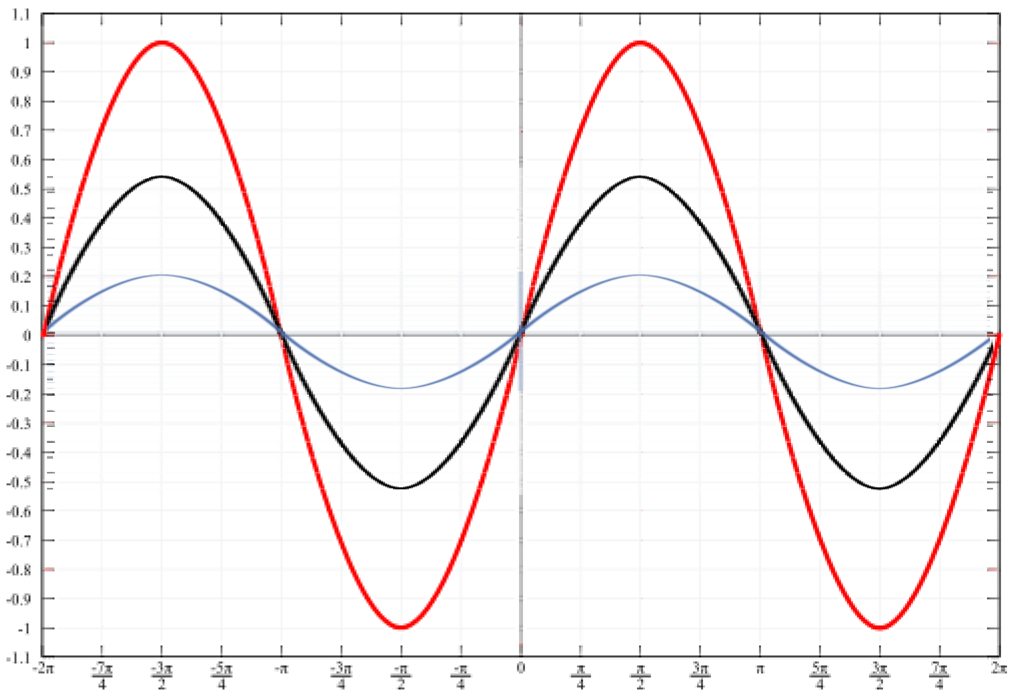


Figure (39) Different amplitudes of an ac sine wave

5.2- AC-AC converters:

A solid-state AC-AC converter converts an AC waveform to another AC waveform, where the output voltage and frequency can be set arbitrarily. The Ac-Ac converters operation method is away off transformers operation method. They do not rely on an induced flux in some sort of windings, they do not have an Iron core. They only contains hard switches and some sort of triggering techniques to operate them.

The hard switches can be triggered in a symmetrical way and for different magnitudes of firing angles to obtain different amounts of output voltages. There are two types of control:

1- ON-OFF control
2- Phase angle control
These methods will be discussed deeply in Special machine subject in 4^{th} stage.

AC voltage controllers are divided into:
1- Single Phase Ac voltage controllers, Half wave and full wave control.
2- Three Phase Ac voltage controllers, Half wave and full wave control.

5.3- Comparison between AC-AC converters and AC transformers:

	AC-AC converters	AC Transformers
Size	Small	Large
Wight	Light	Heavy
Price	Cheap	Expensive
Efficiency	Low	High
Losses type	Switching losses and On-state Losses	Copper losses + Iron losses + Eddy Current and Hysteresis Losses
Drive Requirements	Expensive	None
Maintenance	None	Annually
Output Waveform shape	Parts from a sine wave	Sine wave
Filter requirements	Yes	None

Chapter 6
DC to AC converters

6.1- Introduction:

DC-AC converters are generally called inverters. The function of an inverter is to change a dc input voltage to a symmetric ac output voltage of desired magnitude and frequency. The output voltage waveforms of ideal inverters should be sinusoidal. However, the waveforms of practical inverters are non-sinusoidal and contain certain harmonics.

Inverters are generally classified into two types:

1- Single phase inverters

2- Three phase inverters

6.2- Single phase inverters

6.2.1- Principle of Operation of Half Bridge DC-AC Inverter (R Load)

A single phase Half Bridge inverter is shown in Figure (40). The analysis of the DC-AC inverters is done taking into account the following assumptions and conventions:

1- The current entering node a in Figure (40) is considered to be positive.

2- The switches S1 and S2 are unidirectional, i.e. they conduct current in one direction.

3- The current through S1 is denoted as $i1$ and the current through S2 is $i2$.

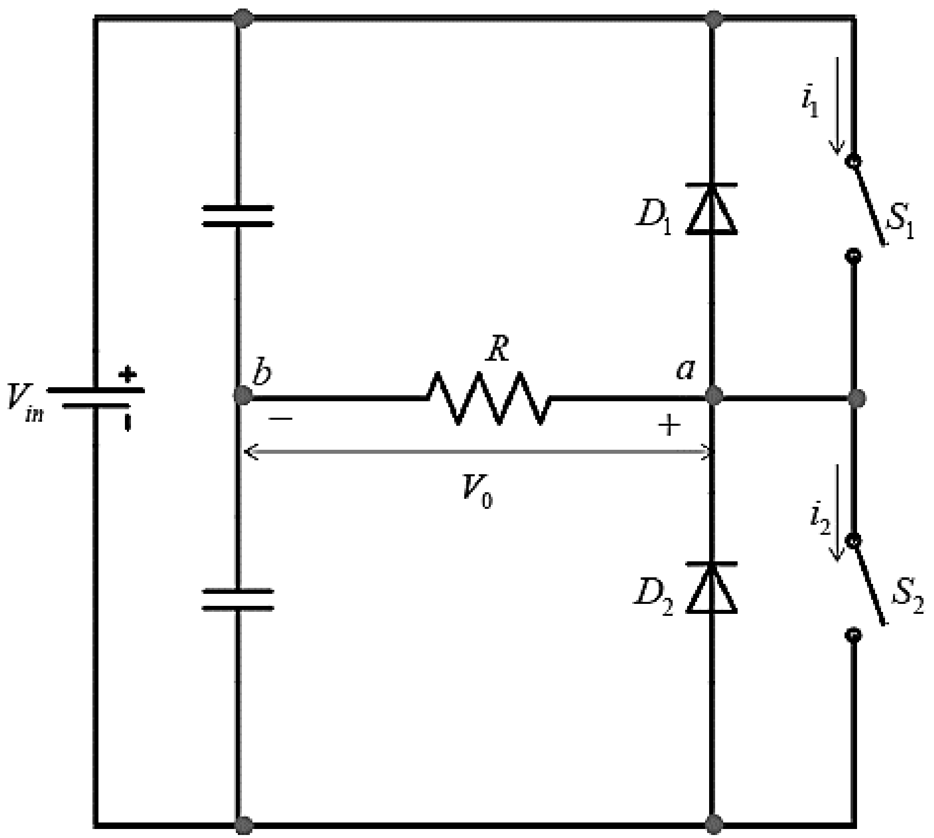


Figure (40) Single phase Half Bridge inverter

The switching sequence is so design Figure (41) that switch S1 is on for the time duration $0 \leq t \leq T1$ and the switch S2 is on for the time duration $T1 \leq t \leq T2$. When switch S1 is turned on, the instantaneous voltage across the load is:

$$v_o = \frac{V_{in}}{2}$$

When the switch S2 is only turned on, the voltage across the load is:

$$v_o = -\frac{V_{in}}{2}$$

The waveform of the output voltage and the switch currents for a resistive load is shown in Figure (41).

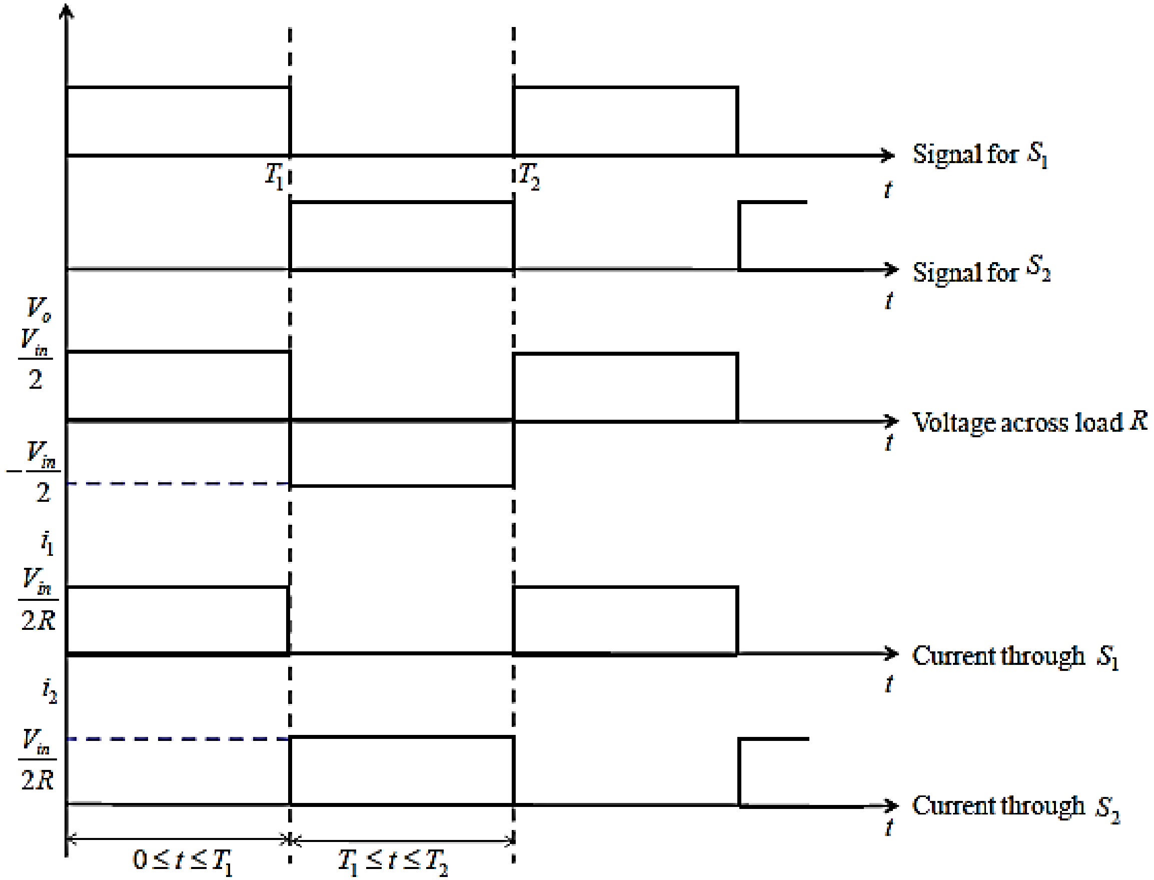


Figure (41) Current and voltage waveforms for DC-AC inverter

The r.m.s value of output voltage is given by:

$$V_{o\,rms} = \left(\frac{1}{T_1}\int_0^{T_1} \frac{{V_{in}}^2}{4}\,dt\right)^{\frac{1}{2}} = \frac{V_{in}}{2}$$

The instantaneous output voltage is rectangular in shape Figure (41). The instantaneous value of Vo can be expressed in Fourier series as:

$$v_o = \frac{a_\circ}{2} + \sum_{n=1}^{\infty} a_n\cos(n\omega t) + b_n\sin(n\omega t)$$

Due to the quarter wave symmetry along the time axis Figure (41), the values of a_n and $a_\circ$ are zero. The value of b_n is given by:

$$b_n = \frac{2V_{in}}{n\pi}$$

Substituting the value of bn

$$v_o = \sum_{n=1,3,5,\ldots}^{\infty} \frac{2V_{in}}{n\pi} \sin(n\omega t)$$

The current through the resistor (iL) is given by

$$i_L = \sum_{n=1,3,5,\ldots}^{\infty} \frac{1}{R}\frac{2V_{in}}{n\pi} \sin(n\omega t)$$

Example 6.1

6.2.2- Single Phase full Bridge DC-AC Inverter with R Load

A single phase bridge DC-AC inverter is shown in Figure (42). The analysis of the single phase DC-AC inverters is done taking into account following assumptions and conventions:

1- The current entering node a in Figure (42) is considered to be positive.

2- The switches S1 , S2 , S3 and S4 are unidirectional, i.e. they conduct current in one direction.

When the switches S1 and S2 are turned on simultaneously for a duration $0 \leq t \leq T1$, the input voltage Vin appears across the load and the current flows from point a to b. If the switches S3 and S4 are turned on for a duration $T1 \leq t \leq T2$, the voltage across the load is reversed and the current through the load flows from point b to a. The voltage and current waveforms across the resistive load are shown in Figure (43). The instantaneous output voltage can be expressed in Fourier series as

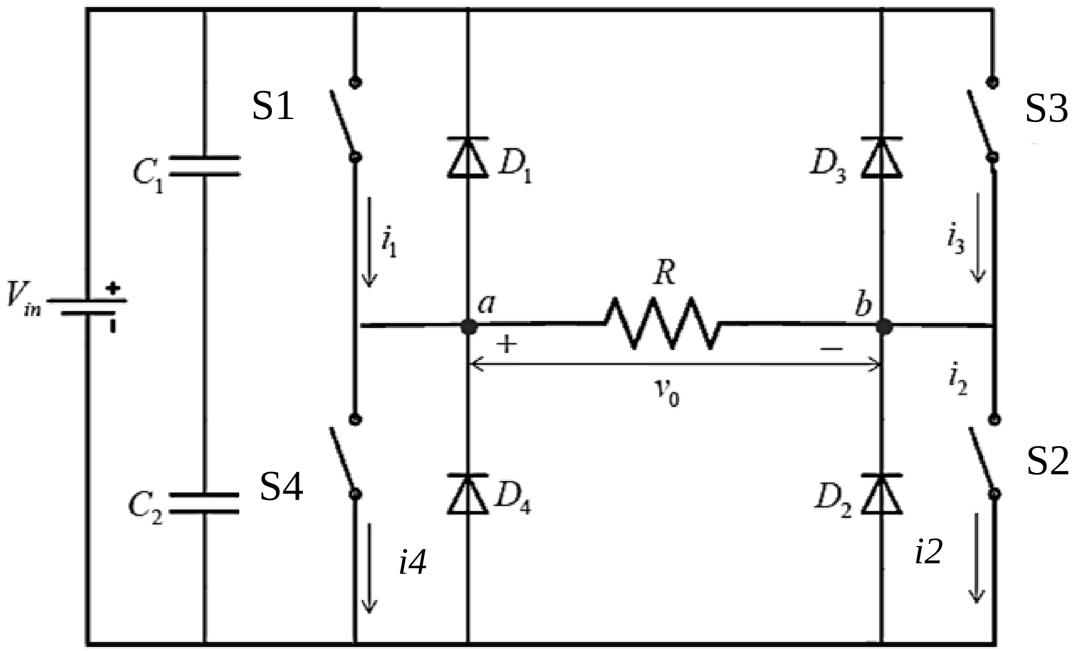


Figure (42) Full bridge DC-AC inverter with resistive load

$$v_o = \frac{a_\circ}{2} + \sum_{n=1}^{\infty} a_n \cos(n\omega t) + b_n \sin(n\omega t)$$

Due to the square wave symmetry along the x-axis as seen in Figure (42), both $a_\circ$ and a_n are zero, and b_n is obtained as:

$$b_n = \frac{4V_{in}}{n\pi}$$

Substituting the value of b_n:

$$v_o = \sum_{n=1,3,5,\dots}^{\infty} \frac{4V_{in}}{n\pi} \sin(n\omega t)$$

The instantaneous current through the resistive load is given by

$$i_L = \sum_{n=1,3,5,\dots}^{\infty} \frac{1}{R}\frac{4V_{in}}{n\pi} \sin(n\omega t)$$

6.3- Three Phase DC-AC Inverters

Three phase inverters are normally used for high power applications. The advantages of a three phase inverter are:

1- The frequency of the output voltage waveform depends on the switching rate of the switches and hence can be varied over a wide range.

2- The direction of rotation of the motor can be reversed by changing the output phase sequence of the inverter.

3- The ac output voltage can be controlled by varying the dc link voltage.

The general configuration of a three phase DC-AC inverter is shown in Figure (43). Two types of control signals can be applied to the switches:

A- 180° conduction

B- 120° conduction

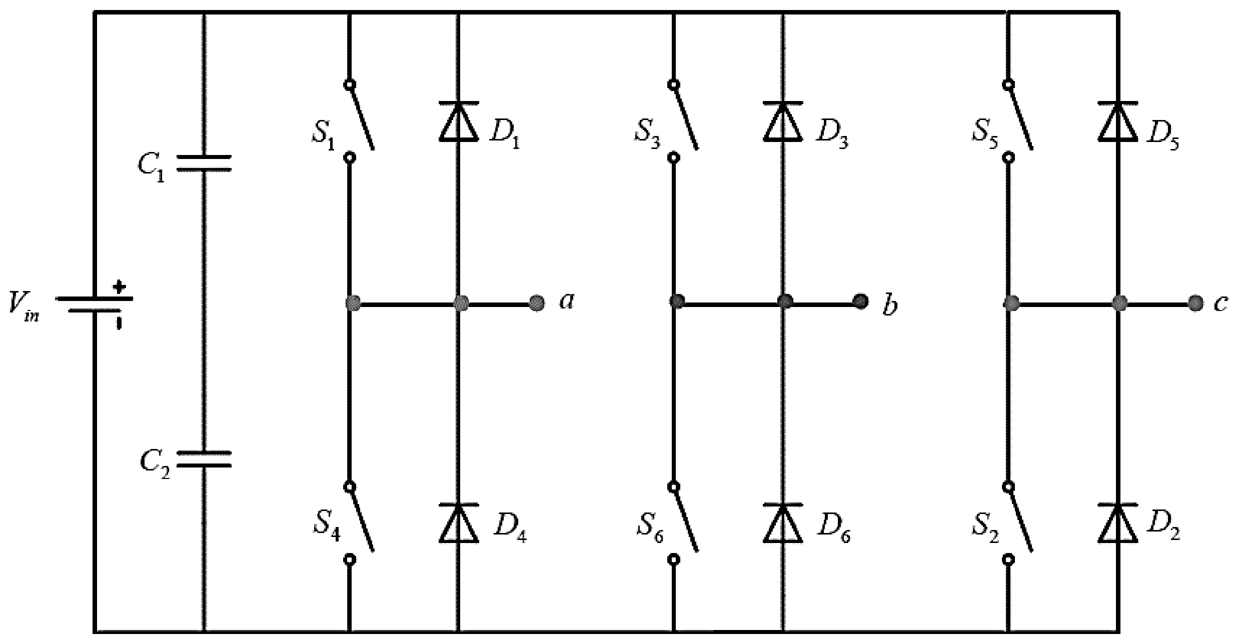


Figure (43) Configuration of a Three-Phase DC-AC Inverter

6.3.1- 180-Degree Conduction with Star Connected Resistive Load

The configuration of the three phase inverter with star connected resistive load is shown in Figure (44). The following convention is followed:

- A current leaving a node point *a*, *b* or *c* and entering the neutral point *n* is assumed to be positive.
- the three resistances are equal, $R_a = R_b = R_c = R$.

In this mode of operation each switch conducts for 180°. Hence, at any instant of time three switches remain on. When S1 is on, the terminal a gets connected to the positive terminal of input DC source. Similarly, when S4 is on, terminal a gets connected to the negative terminal of input DC source. There are six possible modes of operation in a cycle and each mode is of 60° duration and the explanation of each mode is as follows:.

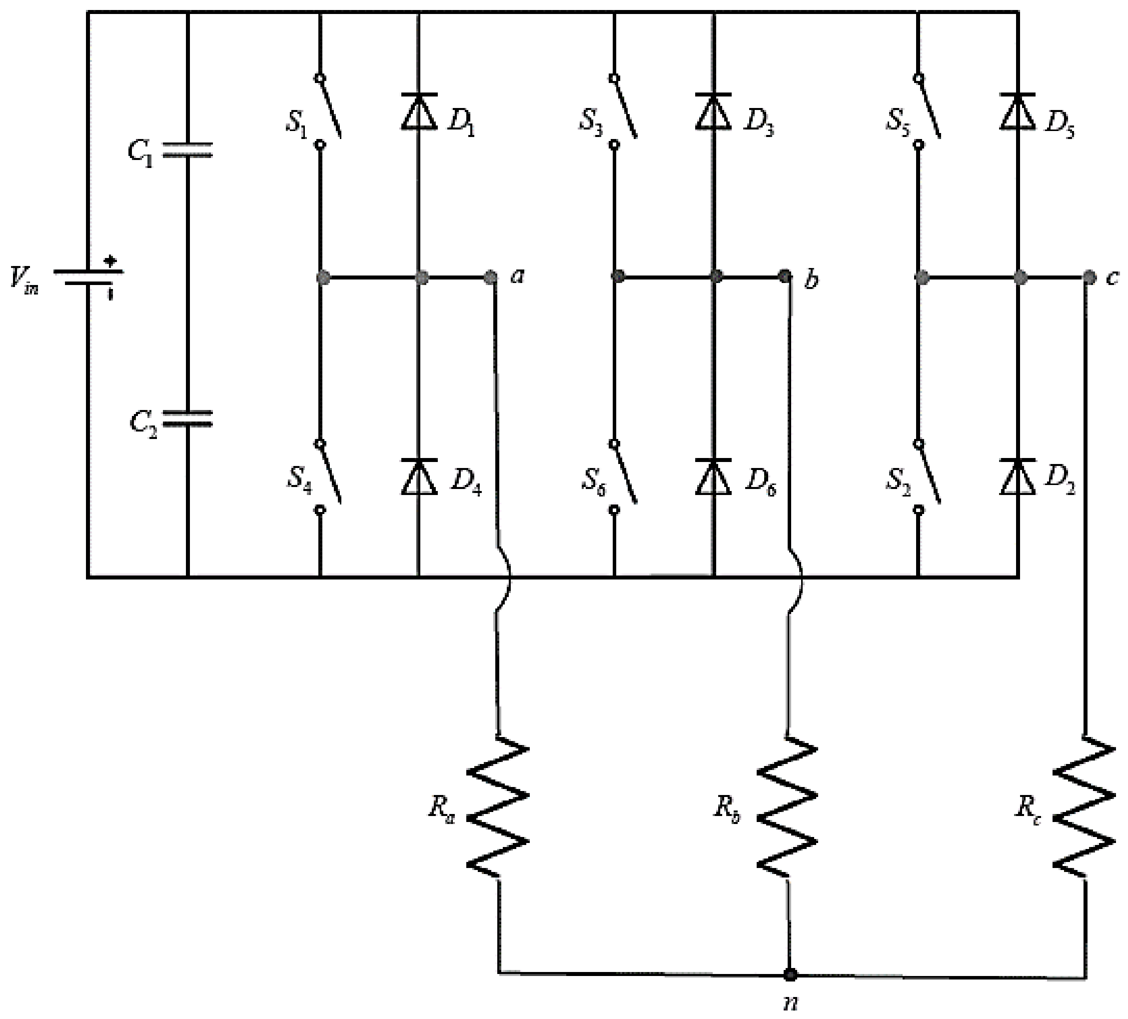


Figure (44) Three-Phase DC-AC Inverter with star connect resistive load

Mode 1: In this mode the switches S5 , S6 and S1 are turned on for time interval $0 \le \omega t \le \frac{\pi}{3}$

As a result of this the terminals a and c are connected to the positive terminal of the input DC source and the terminal b is connected to the negative terminal of the DC source. The current flow through R_a, R_b and R_c is shown in Figure (45-a) and the equivalent circuit is shown in Figure (45-b). The equivalent resistance of the circuit shown in Figure (45-b) is:

$$R_{eq} = R + \frac{R}{2} = \frac{3R}{2}$$

The current i delivered by the DC input source is $i = \frac{V_{in}}{R_{eq}} = \frac{2}{3}\frac{V_{in}}{R}$

The currents i_a and i_b are $i_a = i_c = \frac{1}{3}\frac{V_{in}}{R}$

Keeping the current convention in mind, the current i_b is $i_b = -i = -\frac{2}{3}\frac{V_{in}}{R}$

Having determined the currents through each branch, the voltage across each branch is:

$$v_{an} = v_{cn} = i_a R = \frac{V_{in}}{3} \qquad v_{bn} = i_b R = -\frac{2V_{in}}{3}$$

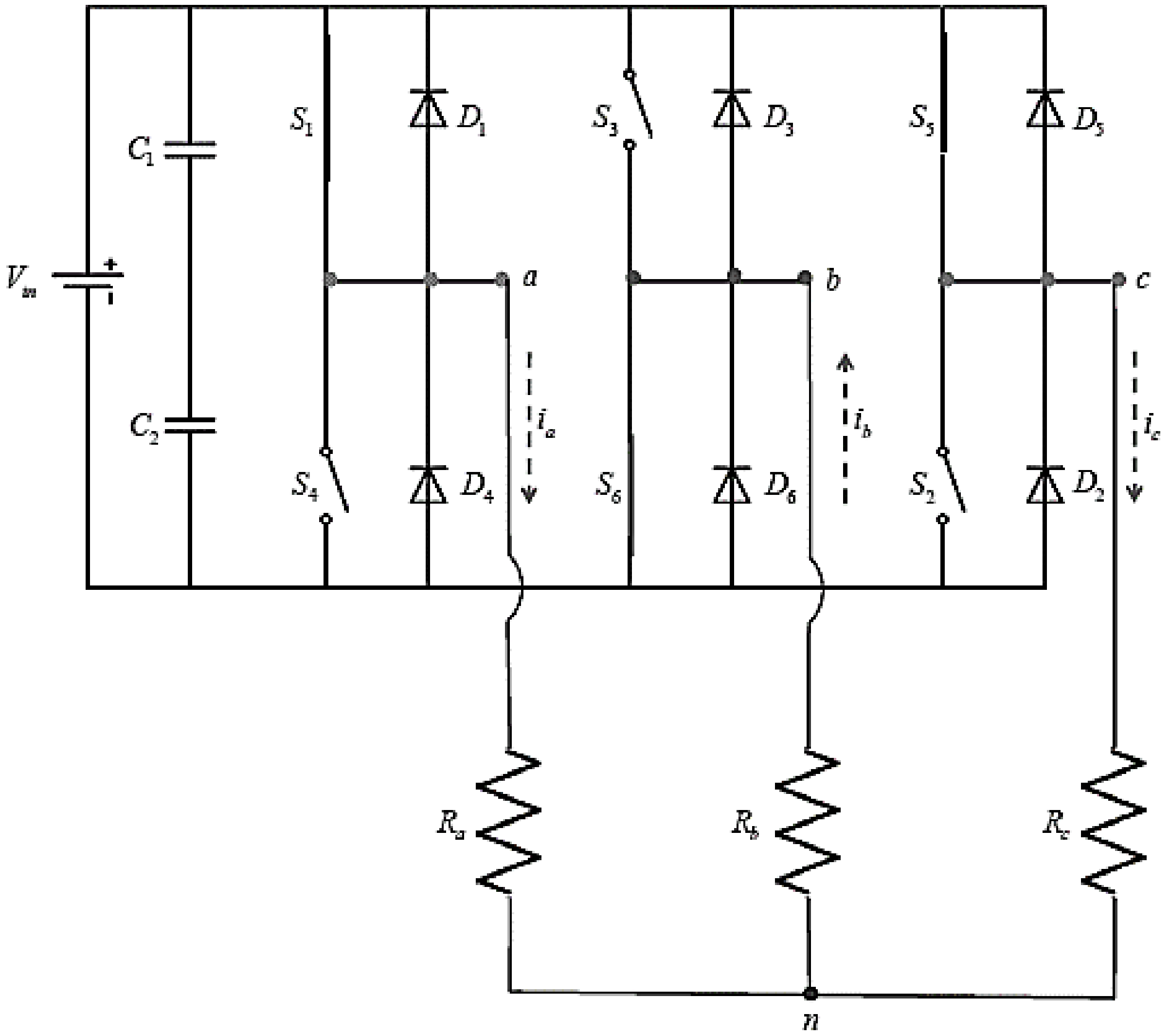


Figure (45-a) Current through the load in Mode 1

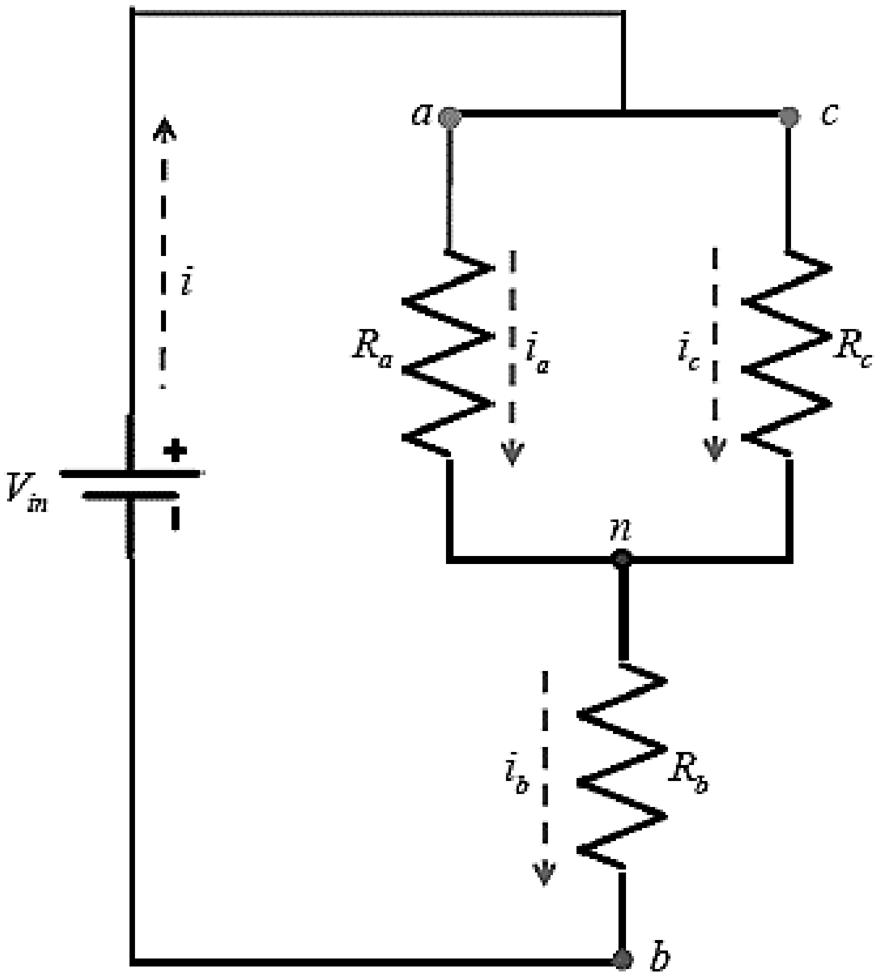


Figure (45-b) Equivalent circuit in Mode 1

Mode 2: In this mode the switches S6 , S1 and S2 are turned on for time interval $\frac{\pi}{3} \leq \omega t \leq \frac{2\pi}{3}$. The current flow and the equivalent circuits are shown in Figure (46-a) and Figure (46-b) respectively. Following the reasoning given for mode 1, the currents through each branch and the voltage drops are given by:

$$i_b = i_c = -\frac{1}{3}\frac{V_{in}}{R}$$

$$i_a = \frac{2}{3}\frac{V_{in}}{R}$$

$$v_{bn} = v_{cn} = -\frac{V_{in}}{3}$$

$$V_{an} = \frac{2V_{in}}{3}$$

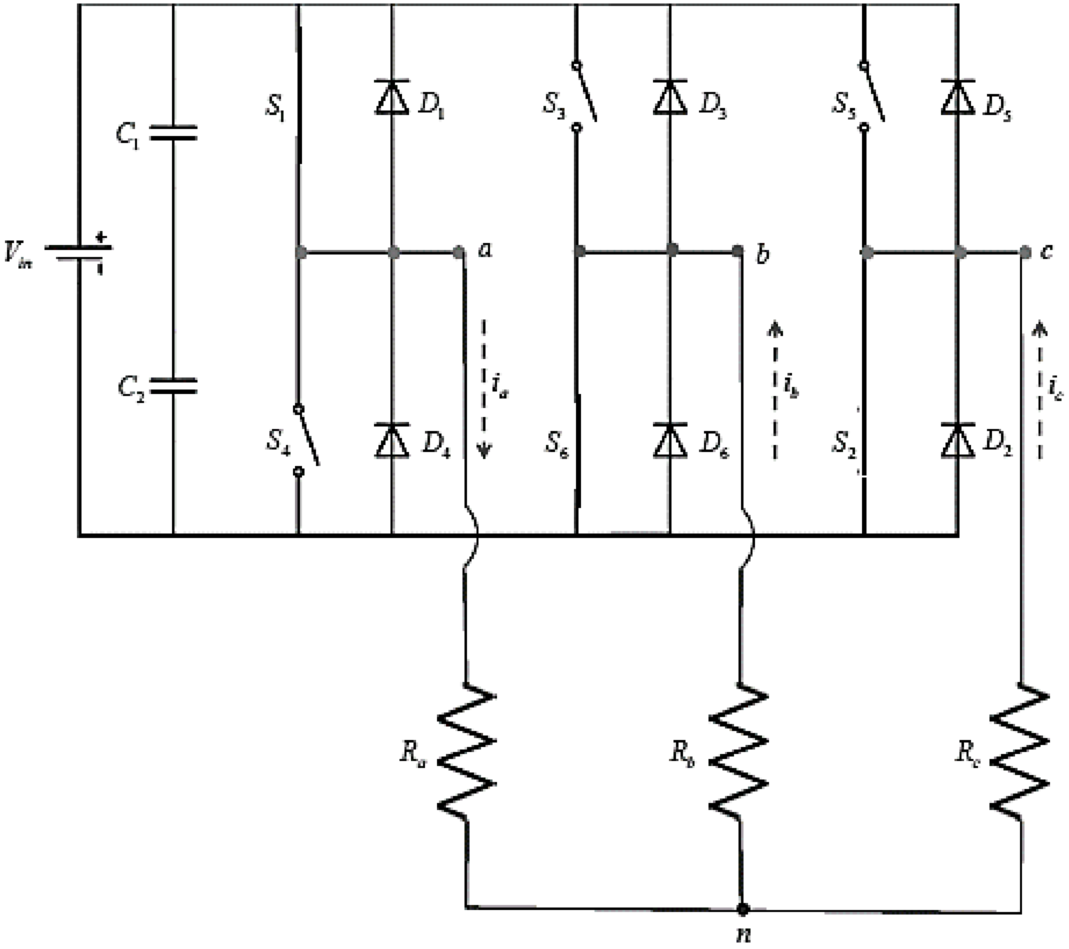


Figure (46-a) Current through the load in Mode 2

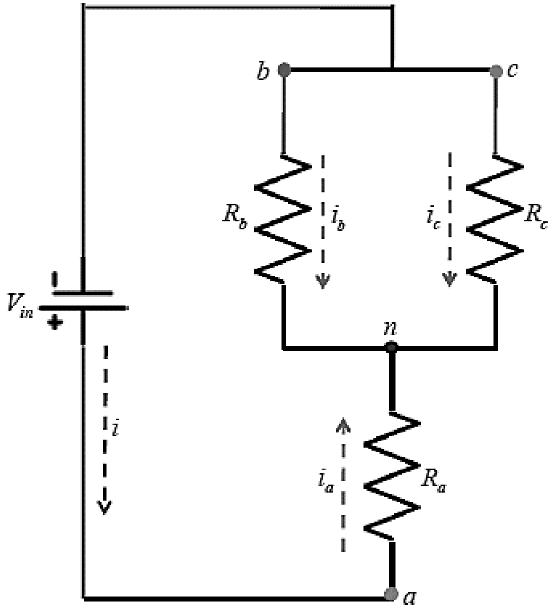


Figure (46-b) Equivalent circuit in Mode 2

Mode 3: In this mode the switches S1 , S2 and S3 are on for $\frac{2\pi}{3} \leq \omega t \leq \pi$. The current flow and the equivalent circuits are shown in Figure (47-a) and figure (47-b) respectively. The magnitudes of currents and voltages are:

$$i_a = i_b = \frac{1}{3}\frac{V_{in}}{R}$$

$$i_c = -\frac{2}{3}\frac{V_{in}}{R}$$

$$v_{an} = v_{bn} = \frac{V_{in}}{3}$$

$$v_{cn} = -\frac{2V_{in}}{3}$$

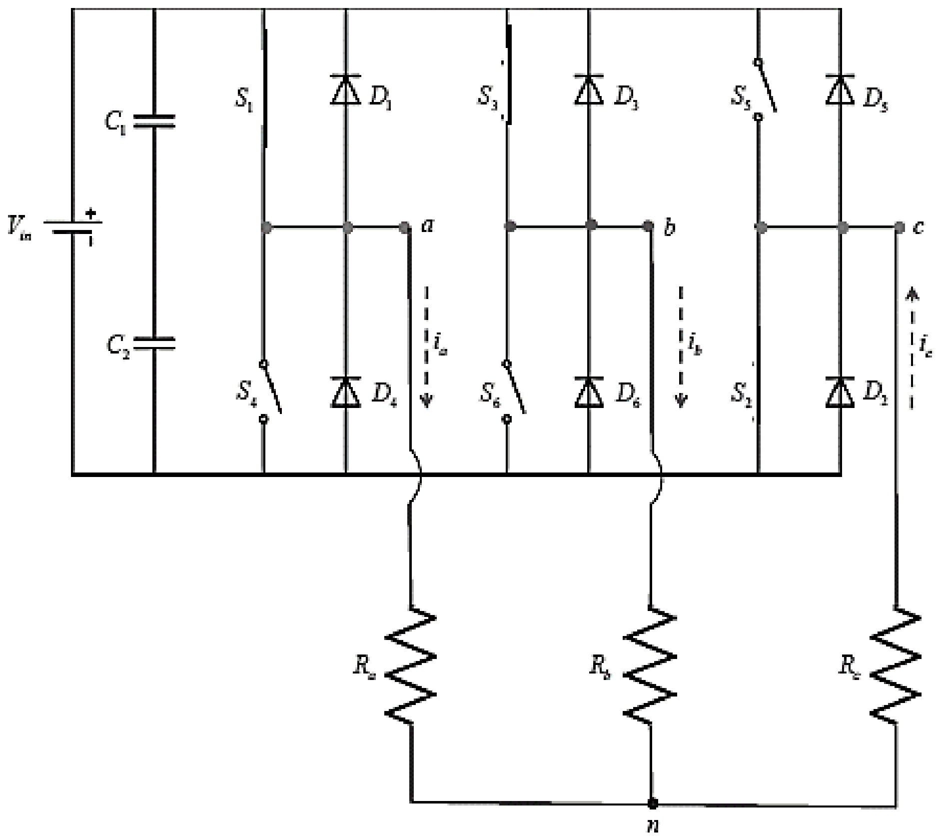


Figure (47-a) Current through the load in Mode 3

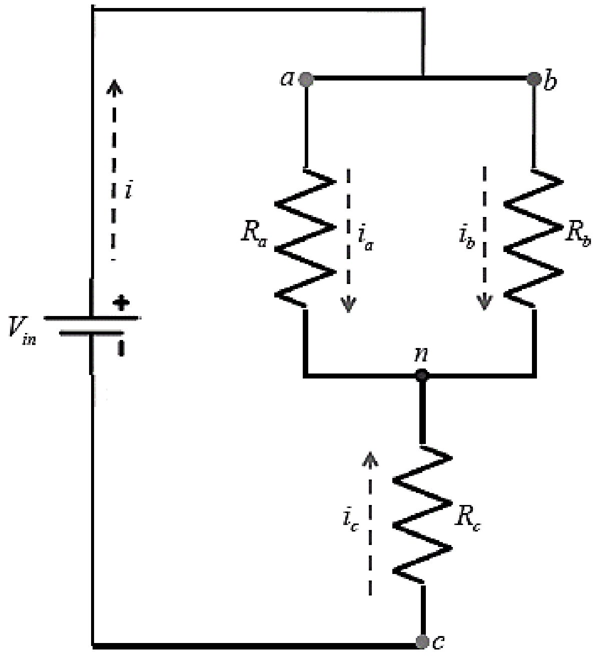


Figure (47-b) Equivalent circuit in Mode 3

For *modes 4*, *5* and *6* the equivalent circuits will be same as *modes 1*, *2* and *3* respectively. The voltages and currents for each mode are shown in table (6.1).

The plots of the phase voltages ($v_{an}, v_{bn}\ and\ v_{cn}$) and the currents ($i_a, i_b\ and\ i_c$) are shown in Figure (48). Having known the phase voltages, the line voltages can also be determined as:

$$v_{ab} = v_{an} - v_{bn}$$
$$v_{bc} = v_{bn} - v_{cn}$$
$$v_{ca} = v_{cn} - v_{an}$$

Example 6.2

Example 6.3

$i_a = i_c = -\frac{1}{3}\frac{V_{in}}{R}$ $v_{an} = v_{cn} = -\frac{V_{in}}{3}$	$i_b = \frac{2}{3}\frac{V_{in}}{R}$ $V_{bn} = \frac{2V_{in}}{3}$	Mode 4
$i_b = i_c = \frac{1}{3}\frac{V_{in}}{R}$ $v_{bn} = v_{cn} = \frac{V_{in}}{3}$	$i_a = -\frac{2}{3}\frac{V_{in}}{R}$ $v_{an} = -\frac{2V_{in}}{3}$	Mode 5
$i_a = i_b = -\frac{1}{3}\frac{V_{in}}{R}$ $v_{an} = v_{bn} = -\frac{V_{in}}{3}$	$i_c = \frac{2}{3}\frac{V_{in}}{R}$ $V_{cn} = \frac{2V_{in}}{3}$	Mode 6

Table (6.1)

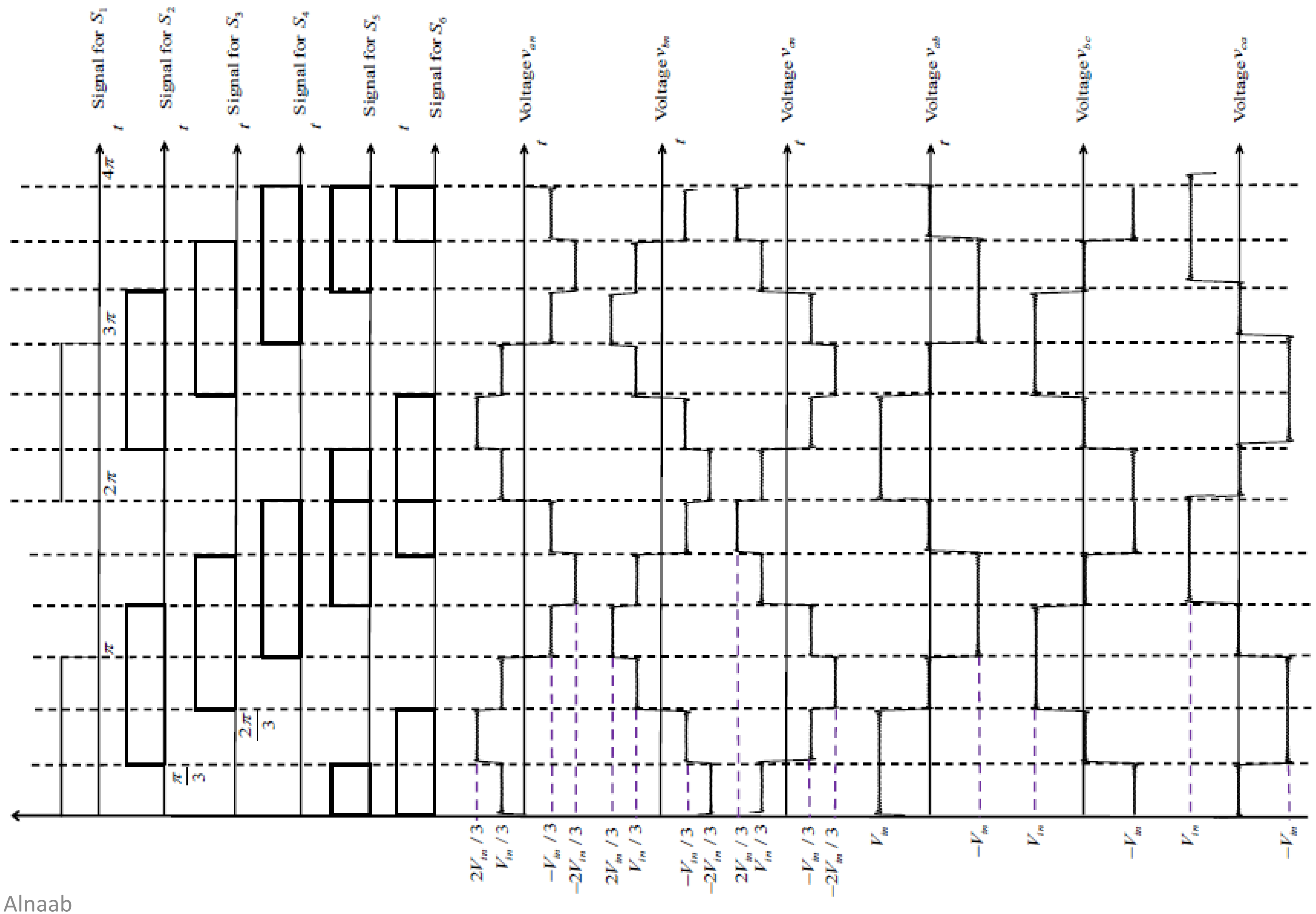


Figure (48) Voltage waveforms for Resistive load for 180°

6.4- Voltage Control of DC-AC Inverters Using PWM

The electric motors used in EV applications are required to have large speed ranges. Large speed ranges can be achieved by feeding the motor with voltages of different frequencies and also different voltage magnitudes. One of the most convenient voltage control technique to generate variable frequency and magnitude voltages is Pulse Width Modulation (PWM).

6.4.1- Pulse Width Modulation Output

Pulse-width modulation (PWM) provides a way to decrease the total harmonic distortion of load current. A PWM inverter output, with some filtering, can generally meet THD requirements more easily than the square wave switching scheme. The unfiltered PWM output will have a relatively high THD, but the harmonics will be at much higher frequencies than for a square wave, making filtering easier.

In PWM, the amplitude of the output voltage can be controlled with the modulating waveforms. Reduced filter requirements to decrease harmonics and the control of the output voltage amplitude are two distinct advantages of PWM. Disadvantages include more complex control circuits for the switches and increased losses due to more frequent switching. Control of the switches for sinusoidal PWM output requires (1) a reference signal, sometimes called a modulating or control signal, which is a sinusoid in this case and (2) a carrier signal, which is a triangular wave that controls the switching frequency. Bipolar and unipolar switching schemes are discussed next.

6.4.2- Bipolar Switching

Figure (49) illustrates the principle of sinusoidal bipolar pulse-width modulation.

Figure (49-a) shows a sinusoidal reference signal and a triangular carrier signal. When the instantaneous value of the sine reference is larger than the triangular carrier, the output is at +Vdc, and when the reference is less than the carrier, the output is at -Vdc:

$$V_o = +V_{dc} \; for \; v_{sin} > v_{triangle}$$
$$V_o = -V_{dc} \; for \; v_{sin} < v_{triangle}$$

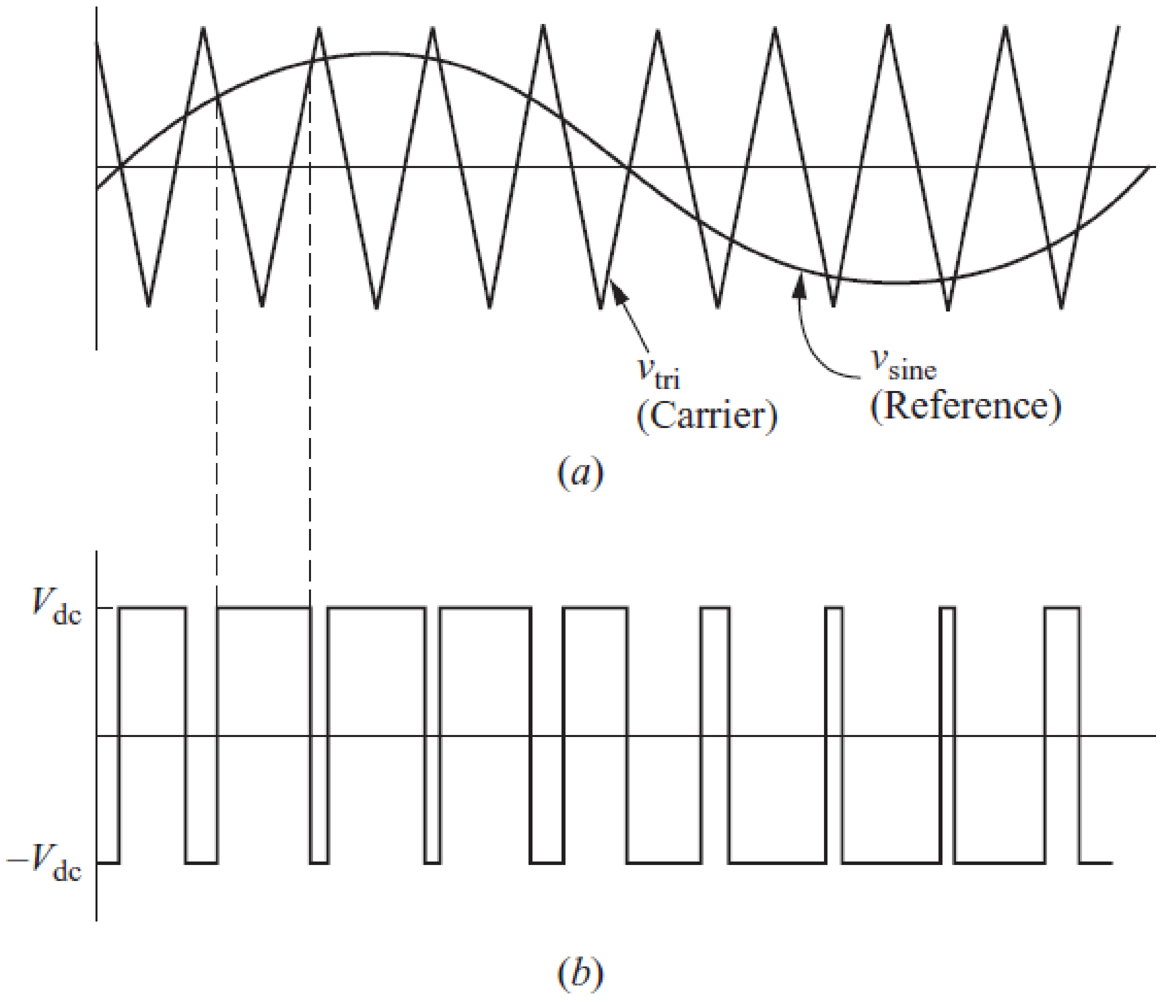


Figure (49) Bipolar pulse-width modulation. (a) Sinusoidal reference and triangular carrier; (b) Output voltage waveform

This version of PWM is bipolar because the output alternates between plus and minus the dc supply voltage.
The switching scheme that will implement bipolar switching using the full-bridge inverter of Figure (50) is determined by comparing the instantaneous reference and carrier signals:

$S1 \text{ and } S2 \text{ are on when } V_o = +V_{dc} \text{ for } v_{sin} > v_{triangle}$
$S3 \text{ and } S4 \text{ are on when } V_o = -V_{dc} \text{ for } v_{sin} < v_{triangle}$

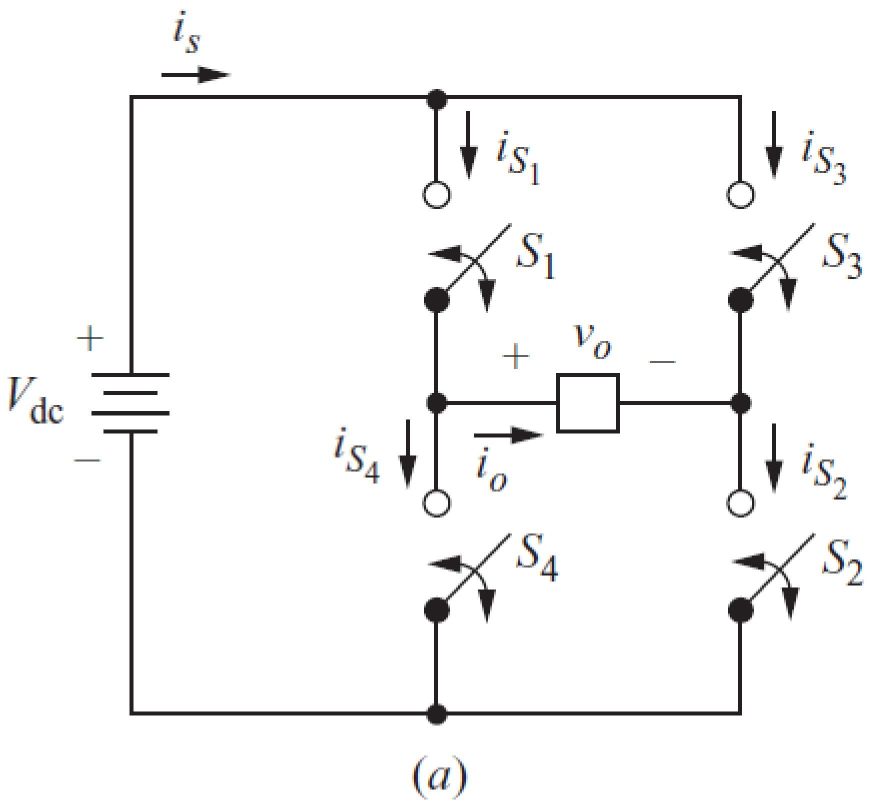


Figure (50) single phase full-bridge inverter

6.4.3- Unipolar Switching

In a unipolar switching scheme for pulse-width modulation, the output is switched either from high to zero or from low to zero, rather than between high and low as in bipolar switching. One unipolar switching scheme has switch controls in Figure (50) as follows:

$S1\ is\ on\ when\ v_{sin} > v_{triangle}$
$S2\ is\ on\ when\ -v_{sin} < v_{triangle}$
$S3\ is\ on\ when\ -v_{sin} > v_{triangle}$
$S4\ is\ on\ when\ v_{sin} < v_{triangle}$

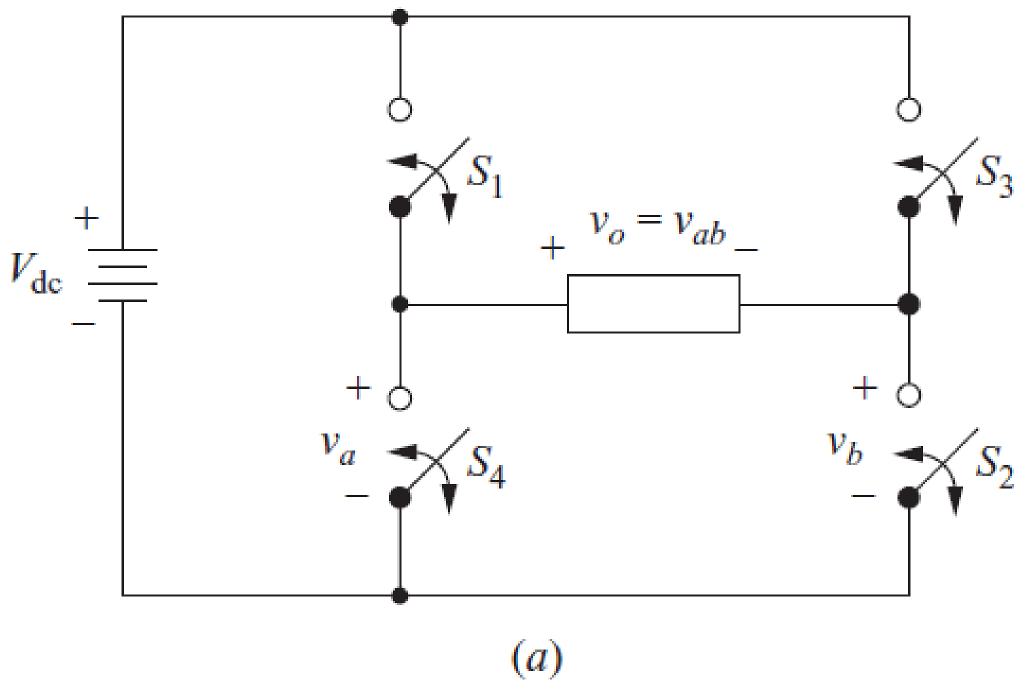


Figure (51-a) Full-bridge converter for unipolar PWM

Note that switch pairs (S1, S4) and (S2, S3) are complementary, when one switch in a pair is closed, the other is open. The voltages va and vb in Figure (51-a) alternate between +Vdc and zero. The output voltage $vo = V_{ab} = v_a - v_b$ is as shown in Figure (51-d).

Another unipolar switching scheme has only one pair of switches operating at the carrier frequency while the other pair operates at the reference frequency, thus having two high-frequency switches and two low-frequency switches. In this switching scheme,

$S1\ is\ on\ when\ \ v_{sin} > v_{triangle}$ (high frequency)
$S4\ is\ on\ when\ \ v_{sin} < v_{triangle}$ (high frequency)

$S2\ is\ on\ when\ \ v_{sin} > 0$ (low frequency)
$S3\ is\ on\ when\ \ v_{sin} < 0$ (low frequency)

where the sine and triangular waves are as shown in Figure (52). Alternatively, S2 and S3 could be the high-frequency switches, and S1 and S4 could be the low frequency switches.

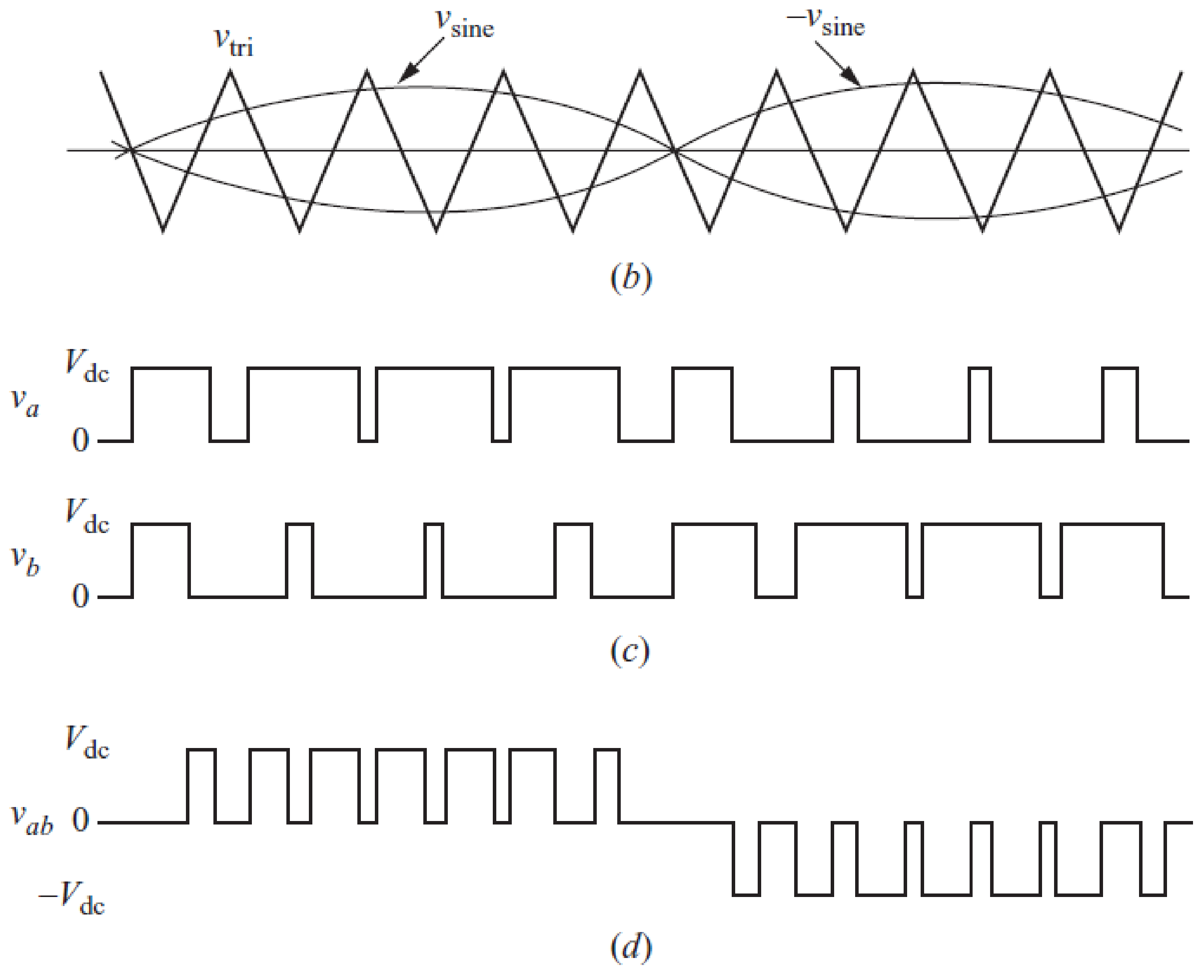


Figure (51) (b) Reference and carrier signals; (c) Bridge voltages va and vb; (d) Output voltage.

6.4.4- PWM DEFINITIONS AND CONSIDERATIONS

1- Frequency modulation ratio mf. The Fourier series of the PWM output voltage has a fundamental frequency which is the same as the reference signal. Harmonic frequencies exist at and around multiples of the switching frequency. The magnitudes of some harmonics are quite large, sometimes larger than the fundamental. However, because these harmonics are located at high frequencies, a simple low-pass filter can be quite effective in removing them. The frequency modulation ratio mf is defined as the ratio of the frequencies of the carrier and reference signals,

$$m_f = \frac{f\ carrier}{f\ reference} = \frac{f tri}{f sine}$$

Increasing the carrier frequency (increasing mf) increases the frequencies at which the harmonics occur. A disadvantage of high switching frequencies is higher losses in the switches used to implement the inverter.

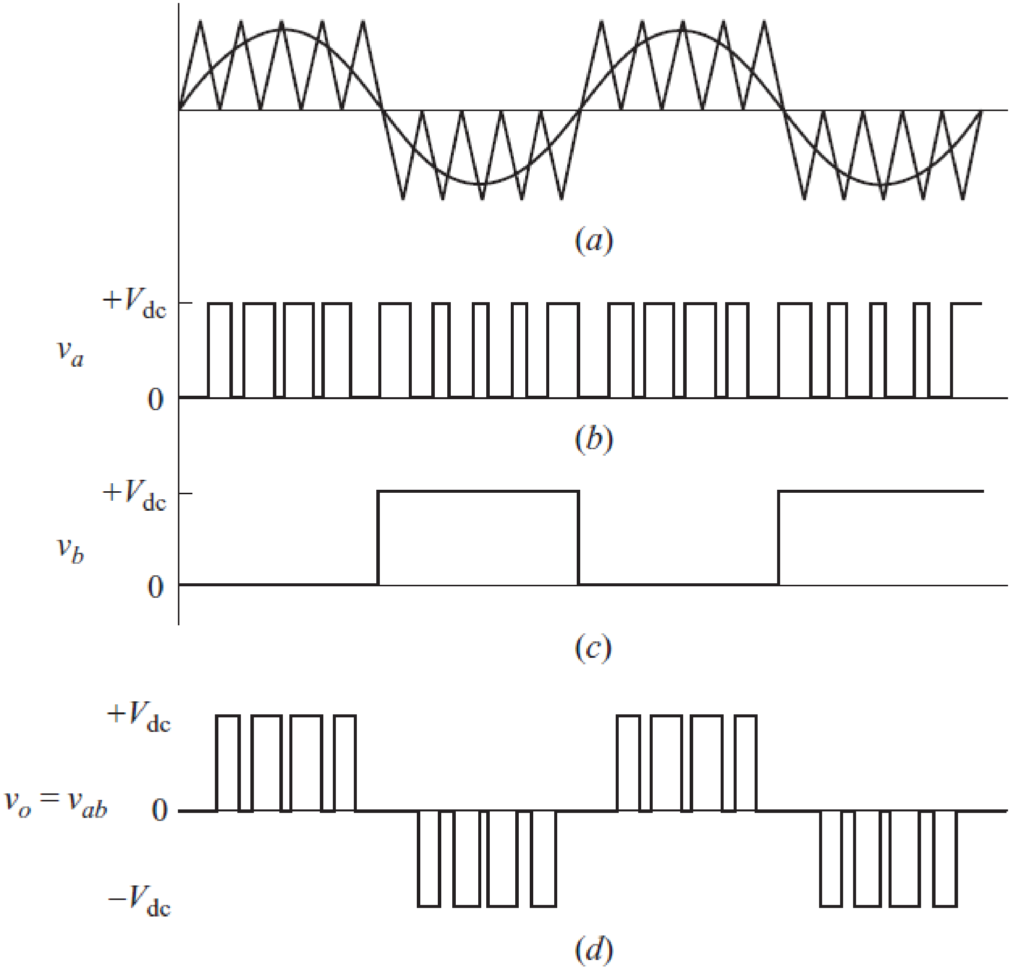


Figure (52) Unipolar PWM with high- and low-frequency switches.

2- Amplitude modulation ratio m_a. The amplitude modulation ratio m_a is defined as the ratio of the amplitudes of the reference and carrier signals:

$$m_a = \frac{Vm, refernece}{Vm, carrier} = \frac{Vm, sine}{Vm, tri}$$

If ma $\leq$ 1, the amplitude of the fundamental frequency of the output voltage V1 is linearly proportional to ma. That is,

$$v_a = m_a V_{dc}$$

The amplitude of the fundamental frequency of the PWM output is thus controlled by m_a. This is significant in the case of an unregulated dc supply voltage because the value of m_a can be adjusted to compensate for variations in the dc supply voltage, producing a constant-amplitude output. Alternatively, m_a can be varied to change the amplitude of the output. If ma is greater than 1, the amplitude of the output increases with m_a, but not linearly.

3- Switches. The switches in the full-bridge circuit must be capable of carrying current in either direction for pulse-width modulation just as they did for square wave operation. Feedback diodes across the switching devices are necessary, as was done in the inverter in Fig. 8-3a. Another consequence of real switches is that they do not turn on or off instantly. Therefore, it is necessary to allow for switching times in the control of the switches just as it was for the square-wave inverter.

4- Reference voltage. The sinusoidal reference voltage must be generated within the control circuit of the inverter or taken from an outside reference. It may seem as though the function of the inverter bridge is unnecessary because a sinusoidal voltage must be present before the bridge can operate to produce a sinusoidal output. However, there is very little power required from the reference signal. The power supplied to the load is provided by the dc power source, and this is the intended purpose of the inverter. The reference signal is not restricted to a sinusoid, and other wave-shapes can function as the reference signal.

What have you learned?